Implicit and Explicit Numerical Solution of Saint-Venent Equations for Simulating Flood Wave in Natural Rivers

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Abstract

River flow predictions are needed in many water resources management. The beginning of the modern study of unsteady flow in open channels can be traced to the latter half of the nineteenth century when the French engineer Saint-Venant introduced the partial differential equations of continuity and momentum governing free surface flow in open channels. These equations are highly nonlinear and therefore do not have analytical solutions. The computer revolution in twentieth century made a new era where numeric methods can be utilized effectively to solve nonlinear partial differential equations. This paper presents the results of two different numerical methods, namely; Preissmann and Lax diffusive schemes for numerical solution of Saint-Venant equations that govern the propagation of flood wave, in natural rivers, with the objective of the better understanding of this propagation process. The results have shown that the hydraulic parameters play important game in the flood wave propagation. The results of these numerical solutions are compared with the HEC-RAS commercial computer model.

Keywords: numerical flood routing, Saint-Venant equations, Preissmann, Lax, HEC-RAS

1. INTRODUCTION

Understanding flood wave routing theory and solving the governing equations accurately is an important issue in hydrology and hydraulics. In unsteady open channel flows, the velocity and water depth change with time and longitudinal position. For one-dimensional applications, the relevant flow parameters (e.g. \( V \) and \( y \)) are functions of time and longitudinal distance. Flood wave propagation in overland and open channel flow may be described by the complete equations of motion for unsteady non uniform flow, known as the dynamic wave equations, first proposed by Saint-Venant in 1871[1]. These equations are nonlinear and therefore do not have analytical solutions. With the greatly improved speed and capacity of digital computers in recent years, dynamic routing models have been widely used for flood forecasting. The first major mathematical model of a river system was developed by J.J. Stoker for the Ohio and Mississippi systems. There have been numerous studies in the literature to solve the Saint-Venant equations by using different numerical techniques. In this research solution of the fully Saint-Venant equations through Lax diffusive explicit scheme and Preissmann implicit scheme for unsteady flow simulation in open channels is presented.

2. GOVERNING EQUATIONS

The dynamic routing model is based on the dynamic wave theory of the Saint-Venant equations which consist of the continuity and momentum equations. For prismatic channels having no lateral inflow or outflow the continuity and momentum equations defined as[2]:

\[
\frac{\partial y}{\partial t} + D_{h} \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x} = 0 \tag{1}
\]

\[
\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} = g(S_0 - S_f) \tag{2}
\]

(Continuity equation)

(Momentum equation)
3. MODEL DEVELOPMENT AND METHODOLOGY

The continuity and momentum equations form a set of nonlinear hyperbolic partial differential equations. A closed form solution of these equations is not available except for very simplified cases. Therefore, numerical methods are used for their integration. The numerical solution of Eqs. 1 and 2 can be obtained if appropriate initial and boundary conditions are prescribed. In this work, the fully Saint-Venant equations are solved for a rectangular wide river with 29 km length using Lax diffusive explicit scheme and four-point Preissmann implicit finite difference scheme. The results of these numerical solutions are compared with the HEC-RAS commercial computer model.

4. LAX DIFFUSIVE SCHEME

We have divided the \( x-t \) plane into a grid that the grid interval along the \( x \)-axis is \( \Delta x \) and the grid interval along the \( t \)-axis is \( \Delta t \). For brevity, we will call the \( i\Delta x \) grid point \( i \) and the \((i + 1)\Delta x \) grid point \( i + 1 \). For the time axis, we will use \( k \) for \( k\Delta t \) grid point and \( k + 1 \) for the \((k + 1)\Delta t \) grid point. To refer to different variables at these grid points, we will use the number of the spatial grid as a subscript and that of the time grid as a superscript. We will denote the known time level by superscript \( k \) and the unknown time level by \( k + 1 \).

To solve the Saint Venant equations, we may select the following finite difference approximations:

\[
\begin{align*}
\frac{\partial f}{\partial x} & \approx \frac{f_{i+1}^k - f_i^k}{2\Delta x} \\
\frac{\partial f}{\partial t} & \approx \frac{f_i^{k+1} - f_i^k}{\Delta t}
\end{align*}
\]  

(3)  

(4)

This finite-difference scheme is inherently unstable. Lax presented a slight variation of the unstable scheme. This scheme is one of the simplest to program and yields satisfactory results for typical engineering applications. In this scheme the partial derivatives and other variables are approximated as follows[2]:

\[
\begin{align*}
\frac{\partial f}{\partial x} & \approx \frac{f_{i+1}^k + f_{i-1}^k}{2\Delta x} \\
\frac{\partial f}{\partial t} & \approx \frac{f_i^{k+1} - f_i^k}{\Delta t} \\
F^* & = \frac{1}{2} (f_{i-1}^k + f_{i+1}^k) \\
D^* & = \frac{1}{2} (D_{i-1}^k + D_{i+1}^k) \\
S^* & = \frac{1}{2} (S_{i-1}^k + S_{i+1}^k)
\end{align*}
\]  

(6)  

(7)  

(8)  

(9)  

(10)

Where, for brevity, we are using \( f \) for both dependent variables, \( y \) and \( V \). We use these approximations in the conservation forms of the governing equations as follows. The conservation form of the governing equations in the matrix form may be written as

\[
U_t + F_x + S = 0
\]  

(11)

In which

\[
U = \left( \begin{array}{c} A^2 \\ A \end{array} \right) \quad F = \left( \begin{array}{c} V^A \\ \nu^2 A^2 + gA \end{array} \right) \quad S = \left( \begin{array}{c} 0 \\ -gA(S_0 - S_f) \end{array} \right)
\]  

(12)

And \( A\bar{y} \) is moment of flow area about the free surface. Substitution of the finite-difference approximations of Eqs. 6, 7, 8, 9 and 10 into Eq. 11 yields

\[
U_i^{k+1} = \frac{1}{2} (U_{i+1}^k + U_{i-1}^k) - \frac{1}{2\Delta x} (F_i^k - F_{i-1}^k) - S^* \Delta t
\]  

(13)

Once the values of \( A \) and \( V_A \) have been determined at the \((k+1)\) time level, we determine the values of variables of interest, \( y \) and \( V \), and then proceed to the next time step.
5. **Preissmann Implicit Scheme**

Several implicit finite-difference schemes have been used for the analysis of unsteady open-channel flows [3-6]. The Preissmann scheme has been extensively used since the early 1960s [7].

The partial derivatives and other coefficients in this method are approximated as follows [2]:

\[
\frac{\partial f}{\partial x} = \beta f(f_{i+1}^{k+1} - f_{i}^{k+1}) + (1 - \beta)(f_{i}^{k} - f_{i}^{k}) \quad (14)
\]

\[
\frac{\partial f}{\partial t} = \frac{f_{i+1}^{k+1} + f_{i}^{k+1} - f_{i}^{k+1} - f_{i}^{k}}{\Delta x} \quad (15)
\]

\[
f = \frac{1}{2} \beta \left( f_{i+1}^{k+1} + f_{i}^{k+1} \right) + \frac{1}{2} \left( 1 - \beta \right) \left( f_{i+1}^{k} + f_{i}^{k} \right) \quad (16)
\]

Where \( \beta \) is a weighting coefficient; \( f \) refers to both \( V \) and \( y \) in the partial derivatives, and \( f \) stands for \( S \), and \( V \) as a coefficient. By selecting a suitable value for \( \beta \), the scheme may be made totally explicit (\( \beta = 0 \)) or implicit (\( \beta = 1 \)).

By substituting the above finite-difference approximations and the coefficients into Eq. 11, and rearranging the terms of the resulting equation, we obtain

\[
U_{i+1}^{k+1} + U_{i}^{k+1} + 2 \frac{\Delta t}{\Delta x} \left( \beta (F_{i+1}^{k+1} - F_{i}^{k+1}) + (1 - \beta) (F_{i}^{k} - F_{i}^{k}) \right) + \Delta t \left( \beta (S_{i+1}^{k+1} + S_{i}^{k+1}) + (1 - \beta) (S_{i}^{k+1} + S_{i}^{k}) \right) = U_{i}^{k} + U_{i}^{k} \quad (17)
\]

In Eq. 17, we have four unknowns, namely, \( V_{i+1}^{k+1}, A_{i+1}^{k+1}, V_{i}^{k+1}, \) and \( A_{i}^{k+1} \). If we write these two equations for each grid point, we have \( 2n \) equations (\( n = \) number of reaches on the channel). We cannot write these equations for the downstream end. However, we have \( 2(n+1) \) unknowns, i.e., two unknowns for each grid point. Thus, for a unique solution we need two more equations. These are provided by the boundary conditions. By applying the Eq. 17 for each node and the boundary conditions we have a set of nonlinear algebraic equations. Here, the nonlinear system of equations have been solved by Newton-Raphson method.

6. **Initial and Boundary Conditions**

Values of depth (\( y \)) and discharge (\( Q \)) at the beginning of the time step are to be specified at all the nodes along the channel as initial conditions. The two boundary conditions required by the model are the inflow discharge hydrograph at the upstream boundary, and the zero-depth at the downstream boundary. In the Lax diffusive explicit scheme, the Eq. 13 may be used at the interior grid points to compute the unsteady flow depth and flow velocity. At the boundaries, however, we cannot use these equations, since there is no grid point outside the flow domain. In this procedure, for explicit schemes we solve the positive characteristic equation simultaneously with the condition imposed by the boundary for the downstream-end condition and the negative characteristic equation with the upstream-end condition for the upstream boundary [2]. Unlike the explicit schemes, we include directly in the system of equations the equations describing the end conditions. In other words, we do not have to use the characteristic equations or the reflection procedures. This is one of the main advantages of the implicit schemes.

7. **Stability**

For the stability of explicit schemes, it is necessary that the Courant number, \( C_n \), is less than or equal to 1, where

\[
C_n = \frac{|V| \sqrt{gD}}{\Delta x/\Delta t} \quad (18)
\]

Thus, the computational time interval depends upon the spatial grid spacing, flow velocity, and celerity, which are functions of the flow depth. Since the flow depth and the flow velocity may change significantly during the computations, it may be necessary to reduce the size of computational time interval for stability. Courant condition must be satisfied at each grid point during every computational interval.
8. APPLICATION OF MODELS

These models are applied to simulate hypothetical flood routing problems in a wide rectangular river. The results are compared to the HEC-RAS computer model.

9. HYPOTHETICAL FLOOD ROUTING IN A WIDE RECTANGULAR RIVER

Flood routing in a 29 km long wide rectangular river with bed slope \( S_0 \) equal to 0.00061, channel width \( B \) is equal to 120 m considered for study. Uniform flow exists initially base flow \( Q_b \) 100 \( m^3/sec \). Friction slope is predicted using Manning’s equation with roughness coefficient \( n \) equal to 0.023. The upstream discharge hydrograph is given by:

\[
Q(t) = \frac{Q_p}{2} \sin \left( \frac{\pi t - \frac{\pi}{2}}{t_p} \right) + \frac{Q_p}{2} + Q_b \quad \text{for} \quad t \leq t_p
\]

\[
Q(t) = \frac{Q_p}{2} \cos \left( \frac{\pi t - \frac{\pi}{2}}{t_b - t_p} \right) + \frac{Q_p}{2} + Q_b \quad \text{for} \quad t_p < t \leq t_b
\]

\[
Q(t) = Q_b \quad \text{for} \quad t > t_b
\]

Where \( t_b \) is the time base equal to 15 hours, \( t_p \) is the time to peak equal to 5 hours and \( Q_p \) is the peak discharge of hydrograph is equal to 200 \( m^3/sec \). The downstream boundary condition is considered by Manning’s equation:

\[
Q = \frac{1}{n} A R^{2/3} S_f^{1/2}
\]

Where \( R = A / P \) is the hydraulic radius; \( A \) is flow area; \( P \) is the wetted perimeter; \( S_0 \) = channel bottom slope. Friction slope is predicted using Manning’s equation:

\[
S_f = \frac{n^2 V^2}{R^{1/3}}
\]

10. Method of solution

For the Lax diffusive scheme, From the known initial conditions and using the discretized governing equations (Eq. 13) and by solution the positive characteristic equation simultaneously with the downstream-end boundary condition (Eq. 20) and the negative characteristic equation with the upstream-end boundary condition (Eq. 19), the values of \( V \) and \( A \), are obtained at all the nodes[2].

For preissmann implicit scheme using the discretizing governing equations (Eq. 17) for each grid point, we have \( 2n \) equations \( (n = \text{number of reaches on the channel that is here equal to 29}) \). We cannot write these equations for the downstream end. However, we have \( 2(n+1) \) unknowns, i.e., two unknowns for each grid point. Thus, for a unique solution we use the Eq. 19 for upstream-end boundary, where

\[
V_1^{k+1} A_1^{k+1} = Q_1^{k+1}
\]

And by using the Eq. 20 for downstream-end boundary, we obtain

\[
V_{n+1}^{k+1} = \frac{1}{n} (A_{n+1}^{k+1}/(B + 2 \frac{Q_{n+1}}{B}))^{2/3} S_f^{1/2}
\]

The system of equations (Eq. 17 for each node and the boundary conditions (Eqs. 23 and 24)) are a set of nonlinear algebraic equations with \( 2(n+1) \) equations in \( 2(n+1) \) unknowns. We have used the Newton-Raphson method for solving the nonlinear system of equations. In this method, we have

\[
x_{1}^{p+1} = x_{1}^{p} - w^{-1}(x_{0}^{p}) f(x_{0}^{p})
\]

Where \( x_1 \) is the column matrix of 60 unknowns (\( V \) and \( A \) for each grid points) in advanced time step; \( x_0^p \) is the column matrix of initial guess for the unknown parameters, where the superscript \( p \) is the number of iterations \( (p = 0,1,2,...) \); \( w \) is the jacobian square matrix of functions that are generated from Eqs. 17, 22 and 23; \( f \) is the column matrix of functions that are generated from Eqs. 17, 22 and 23. Thus, the matrix form of Eq. 24 may be written as

The preissmann scheme is unconditionally stable provided \( 0.5 \leq \beta \leq 1 \), i.e., the flow variables are weighted towards the \( k + 1 \) time level[2].
respectively, at downstream-end. The peak flow depth 1.60, 1.66, 1.41 m, computed with Lax diffusive, Preissmann scheme and two hours earlier than the downstream-end, the arrival time of peak flow in both Lax diffusive and Preissmann numerical models is.

we apply the correction and iterate the procedure. Here, we take the tolerance equal to the specified tolerance, then we proceed to the next step after applying the correction. Otherwise, we check that from 2 to 2m-2 and subscript j from 1 to m(m is the number of cross sections), using the Eq. 17 we obtain

And for even value of subscript i from 2 to 2m-2 and subscript j from 1 to m, using the Eq. 17 we obtain

And by using the Eq. 23 for downstream-end boundary, we have

Now, we check that \( \sum_{i=1}^{n+1} |\Delta A_i| + |\Delta V_i| \leq \varepsilon \), where \( \varepsilon \) is the specified tolerance. If the sum of the corrections is less than the specified tolerance, then we proceed to the next step after applying the correction. Otherwise, we apply the correction and iterate the procedure. Here, we take the tolerance equal to \( 10^{-4} \). Note that matrix \( w \) contains a large enough percentage of zeros. In the MATLAB software, we can convert the matrix \( w \) to a column matrix using \( \text{sparse} \) command. We may utilize this fact while solving Eq. 45, since a vector solution routine requires less storage and gives more accurate results.

11. RESULTS AND DISCUSSIONS

The initial condition for all of the models corresponds to uniform flow with discharge 100 m³/sec and flow depth 0.86 m. Also, the friction slope is computed using Manning’s equation with roughness coefficient n equal to 0.023. The upstream discharge hydrograph (Eq. 19) and the downstream stage-discharge relationship (Eq. 20) are used as boundary conditions for the models. The computed unsteady flow data through numerical methods and HEC-RAS model at the 16km section are given in Table 1. From table 1 it can be seen that at 16km section, the arrival time of peak flow in both of Lax diffusive and Preissmann numerical models is one hour earlier than the HEC-RAS model and the peak flow discharge 2.38, 2.47, 2.01 m³/s m⁻¹ m width and the peak flow depth 1.63, 1.67, 3.06 m, computed with Lax diffusive, Preissmann scheme, HEC-RAS, respectively at 16km section. The computed unsteady flow data through the numerical methods and HEC-RAS model at the downstream-end are given in Table 2. From table 2 it can be seen at the downstream-end, the arrival time of peak flow in both Lax diffusive and Preissmann numerical models is two hours earlier than the HEC-RAS model and the peak flow discharge 2.31, 2.44, 1.88 m³/s m⁻¹ m width and the peak flow depth 1.60, 1.66, 1.41 m, computed with Lax diffusive, Preissmann scheme and HEC-RAS, respectively, at downstream-end.
Table 1. computed stage and discharge hydrograph at 16km section using all of the models

<table>
<thead>
<tr>
<th>Time (hour)</th>
<th>upstream-end</th>
<th>15 km from upstream end</th>
<th>HEC-RAS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>q (m³/s·m⁻¹)</td>
<td>y (m)</td>
<td>q (m³/s·m⁻¹)</td>
</tr>
<tr>
<td>0</td>
<td>0.83</td>
<td>0.86</td>
<td>0.83</td>
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<tr>
<td>1</td>
<td>0.99</td>
<td>0.95</td>
<td>0.83</td>
</tr>
<tr>
<td>2</td>
<td>1.40</td>
<td>1.17</td>
<td>0.88</td>
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<tr>
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<td>1.92</td>
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<td>1.32</td>
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<td>1.18</td>
<td>1.07</td>
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<td>0.86</td>
<td>0.86</td>
</tr>
<tr>
<td>20</td>
<td>0.83</td>
<td>0.86</td>
<td>0.85</td>
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Table 2. computed stage and discharge hydrograph at downstream-end using all of the models

<table>
<thead>
<tr>
<th>Time (hour)</th>
<th>upstream-end</th>
<th>at downstream-end</th>
<th>Lax diffusive</th>
<th>Preissmann</th>
<th>HEC-RAS</th>
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<td></td>
<td>q (m³/s·m⁻¹)</td>
<td>y (m)</td>
<td>q (m³/s·m⁻¹)</td>
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12. CONCLUSIONS

In this research the solution of fully Saint-Venant equations (continuity and momentum equations) for hypothetical flood routing problems in a wide rectangular river through explicit and implicit numerical
schemes and HEC-RAS computer model is presented. From the results of the numerical methods that were applied in this study, it can be seen that there is a reasonably good matching in the model results, but there is a poor matching between these results and outputs of HEC-RAS computer model. It might be because of momentum equation which is replaced by energy equation in the HEC-RAS model [8]. The magnitude of the wave attenuation that measured with HEC-RAS model is equal to 25% at downstream-end, and it is equal to 3% to 8% in other models. Flow depths that were computed with HEC-RAS model at 15km after the upstream is more than the other models and it is lower than the other models at the downstream-end. Results show that the arrival time of peak flow in the numerical models is one hour earlier than the HEC-RAS model at 15km after upstream and it is equal to two hours at downstream-end.

13. REFERENCES


