Subgrid-scale model effect on large-eddy simulation of a fully developed turbulent duct flow

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Abstract

Large-eddy simulation (LES) of turbulent flow in a square duct is performed with the dynamic Smagorinsky (DS) subgrid-scale (SGS) model at $Re_\tau = 180$. *To investigate the effect of the SGS model on LES predictions, a simulation without an SGS model is also performed and the results are compared with those of the DS model and a reference direct numerical simulation (DNS) data. Simulations are carried out using a second-order finite volume Navier—Stokes solver. LES predictions with the DS model show a reasonable agreement with the DNS data for the mean velocity and Reynolds stresses. An appreciable improvement in LES predictions is also observed compared with the no SGS model predictions.*

Key words: large-eddy simulation, square duct, turbulent flow, dynamic Smagorinsky model.

1. Introduction

Turbulent duct flows are found in many industrial applications such as heating, ventilation, combustion chambers and sustainable drainage systems. These flows also present a unique feature, which is the existence of secondary flow consisting of four pairs of counter-rotating vortices normal to the streamwise direction, which distribute symmetrically about the bisectors of the walls and the diagonals of the square cross-sections [1,2]. This complexity of the flow physics makes an interesting test case for LES.

Turbulent duct flows have been studied previously using DNS [1-4] and LES [5,6]. Due to the long turbulent structures present in a duct flow, a long computational domain is necessary for numerical simulations. Hence, DNS and LES of turbulent duct flow is computationally expensive. Thus, compared to other canonical turbulent flows, fewer numerical simulations exist in the literature and there is a need for further examination of this test case.

In this study, LES is performed for a square duct with a friction Reynolds number $Re_\tau = 180$ using the dynamic Smagorinsky (DS) subgrid-scale (SGS) model. To investigate the effect of the SGS model, a simulation is performed without an SGS model and the results are compared those of the DS model and a reference DNS data [1].

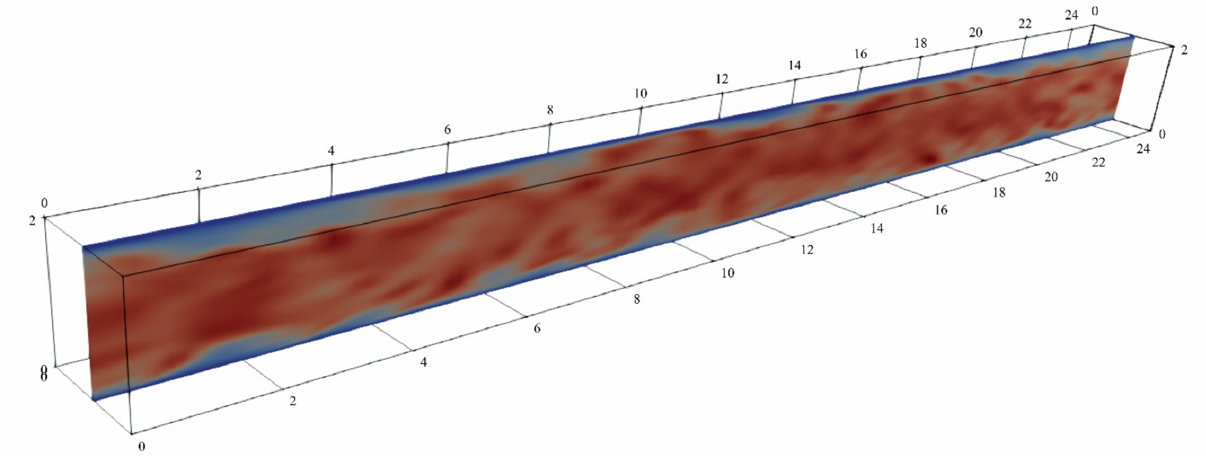

Figure 1: Schematic of the duct flow simulation set-up with a contour plot of the instantaneous streamwise velocity.

2. Governing equations and subgrid-scale model description

The governing equations are obtained by filtering the incompressible Navier-Stokes and continuity equations [7]:

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + F_i, \qquad (1)$$

$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0, \qquad (2)$$

where $\tilde{u}_i$ is the filtered velocity vector, $\tilde{p}$ is filtered pressure, F_i is body force vector and τ_{ij} is the SGS stress tensor, which has to be modeled and is defined as:

$$\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j. \qquad (3)$$

The summation convention is used over the repeated indices in the equations. The dynamic Smagorinsky model [8] with modifications of Lilly [9] is used to close the equations, which has the following formulation

$$\tau_{ij} - \frac{2}{3} K^{SGS} \delta_{ij} = -2C\tilde{\Delta}|\tilde{S}|\tilde{S}_{ij}, \qquad (3)$$

$$\tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right), [\tilde{S}] = \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}}, \qquad (2)$$

where δ_{ij} is the Kronecker delta, $\tilde{\Delta} = 2\sqrt[3]{\Omega}$ (Ω is the volume of a grid cell), K^{SGS} is the SGS turbulent kinetic energy and C is the Smagorinsky coefficient which is dynamically computed.

4. Numerical Method

An unstructured collocated finite volume solver is used for preforming the computations [10]. The conservative form of the incompressible Navier–Stokes equations is solved by using a second-order central differencing in space and a second-order Crank–Nicolson scheme in time. The pressure–velocity coupling is based on a SIMPLEC algorithm with Rhie and Chow interpolation to avoid odd–even oscillations.

Table 1: Specifications of the numerical simulations. The grid spacings in the x,y and z directions are given in wall units: Δx^+, Δy^+ and Δz^+, respectively. The number of grid points are N_x, N_y and N_z in the respective directions. Case 3 is the reference data [1]. The duct half height is denoted by h. *DS denotes the dynamic Smagorinsky model.

Case	Model	Domain size	$N_x \times N_y \times N_z$	Δx^+	Δy^+_{max}	Δz^+_{max}	Re_τ
1	DS*	25h × 2h × 2h	128 × 65 × 65	35	11	11	142.1
2	No subgrid-scale			28	9	9	141.8
3	DNS		= 28 million	10	5	9	148.3

Specifications of the simulations along with those of the reference DNS [1] are given in Table 1. Simulations are performed with a constant bulk velocity and the bulk Reynolds number is $Re_b = 2800$, which is the same as the reference DNS. The grid is uniform in the streamwise direction, whereas a tangent-hyperbolic distribution of grid-points is used in the other directions. Schematic of the duct flow is shown in Figure 1 along with a contour plot of the instantaneous streamwise velocity. The domain is $25h$ long in the x direction, where h is the duct half height. The duct has an aspect ratio of one.

2.4. Results and Discussion

Mean velocity profiles are shown in Figure 2. The mean velocity profile for case 1, with the DS model, agrees well with the DNS results, whereas case 2 under-predicts the mean velocity profile outside the buffer layer. The under-prediction in case 2 is partly due to the over-prediction of the wall shear, as can be seen in Table 1 (Re_τ predictions).

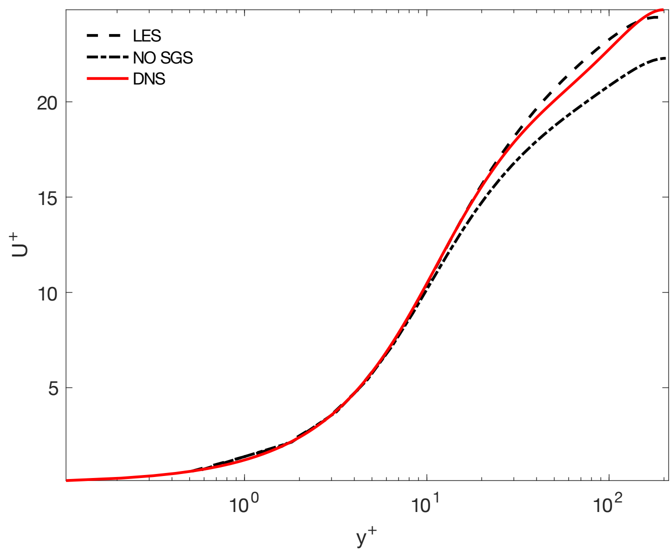

Figure 2: Mean velocity profiles in wall units.

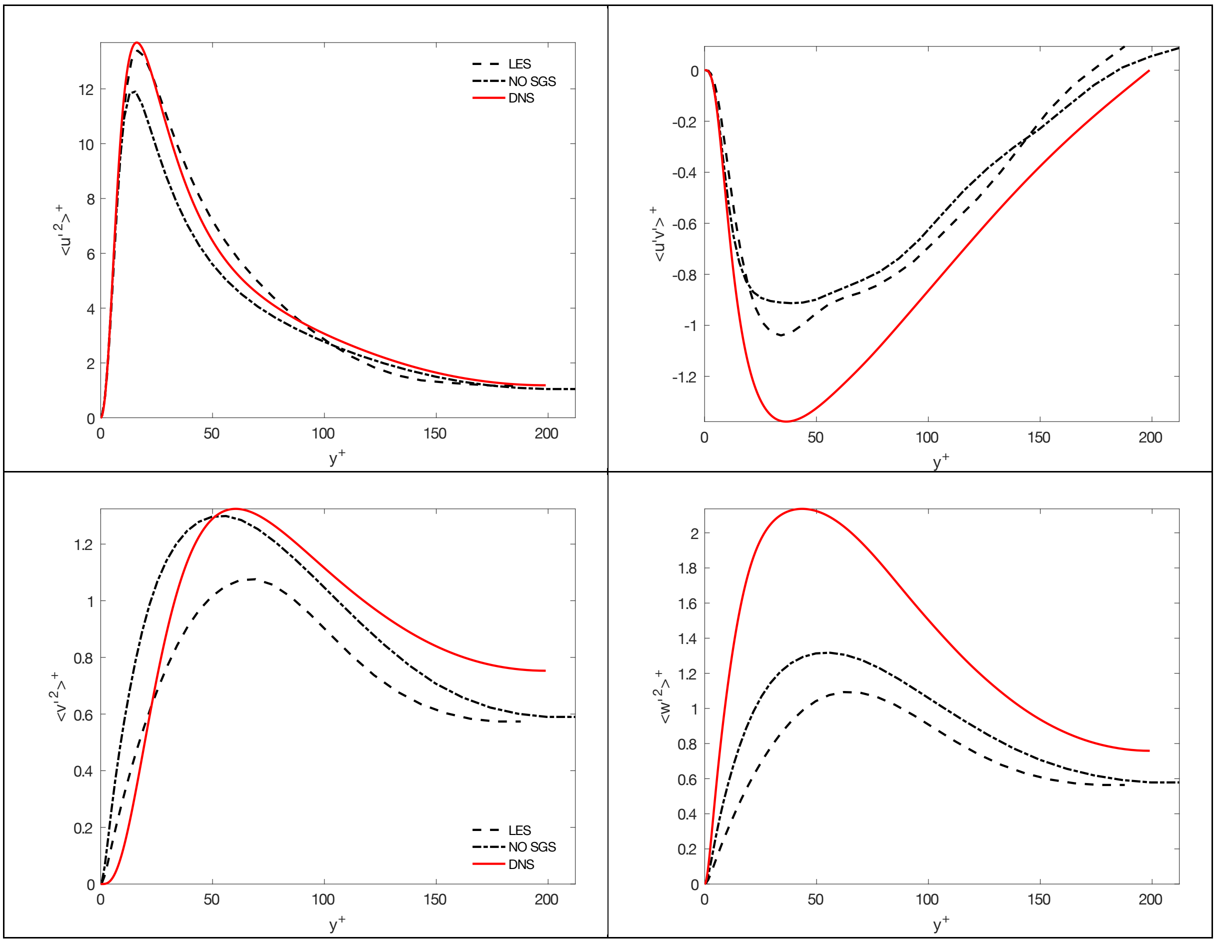

Figure 3: Mean Reynolds stresses in wall units.

Figure 3 shows the mean Reynolds stresses. The streamwise normal Reynolds stress in wall units $< u'u' >^+$ is well-predicted by the DS model, whereas it is largely under-predicted without an SGS model, which is due to the lack of SGS dissipation. This behavior is contrary to what is observed in LES of turbulent channel flow [11]. The peak of $< u'u' >^+$ in case 1 and 3 is around $y^+ = 19$, whereas it is at less y^+ values in case 2.

The $< v'v' >^+$ component of the Reynolds stress is under-predicted in case 2 whereas its peak is well-predicted in case 1, but the near-wall behavior is miss-predicted in both cases. The $< w'w' >^+$ is under-predicted in both cases, but it is better predicted in case 1.

The Reynolds shear stress is slightly better predicted in case 1, indicating that the SGS dissipation is better predicted. The lack of SGS dissipation in case 2 results in a larger misprediction of the SGS shear stress.

3. Conclusions

Large eddy simulations of turbulent duct flow with aspect ratio 1 is performed with the dynamic Smagorinsky (DS) and without a subgrid-scale (SGS) model and the results were compared with the reference direct numerical simulation (DNS) data. It was observed that the DS model predictions were in reasonable agreement with the DNS data, for the mean velocity and Reynolds stresses. Simulations without an SGS model showed under-predictions of the mean velocity and Reynolds stresses. Further investigation of SGS model effects on LES predictions of the turbulent duct flow statistics is necessary.

Acknowledgements

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