Effect of magnetic field direction on MHD turbulent channel flow at low magnetic Reynolds number

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Abstract

Direct numerical simulations (DNSs) of magnetohydrodynamic (MHD) turbulent channel flow at $Re_\tau = 180$ *are performed using a pseudo-spectral Navier—Stokes solver. Simulations are performed in the limit of low magnetic Reynolds number. Three cases are considered, where the magnetic field vector is directed in x, y and z directions. The significance of the magnetic field direction on the mean flow statistics are presented in comparison with the DNS data of a plane turbulent channel flow.*

Key words: direct numerical simulation, magnetohydrodynamic flow, turbulent channel.

1. Introduction

Magnetohydrodynamic (MHD) flows occur in a number of industrial applications such as continuous casting of liquid steel, crystal growth, arc welding, aluminum reduction cells, MHD pumps, flow-meters, generators, propulsion devices, to mention but a few. Other applications of MHD flows include astrophysical and geophysical flows [1].

Several studies have been performed to numerically explore MHD turbulent channel flows at different Reynolds numbers, using direct numerical simulation (DNS) [2-4], large-eddy simulation [6-8] and Reynolds-averaged Navier—Stokes equations (RANS) [9]. In these studies, the effect of the magnetic field on the turbulent flow are studied for the flow in a channel, where it has been found that the magnetic field could significantly modify turbulence characteristics [5].

In this study, DNSs of MHD turbulent channel flow at $Re_\tau = 180$ are performed, in the limit of a low magnetic Reynolds number, with different orientations of the magnetic field. The significance of the direction of the magnetic field on the mean velocity profiles and Reynolds stresses are compared with those of a reference DNS data. A highly accurate pseudo-spectral Navier—Stokes solver is employed in the simulations. For simplicity, only three cases are shown and a limited number of statistics are presented. The results, presented here, are in excellent agreement with previous studies. Further analysis has also been carried out on the structure of the turbulent field and the higher-order moments of the velocity, which will be presented at the conference.

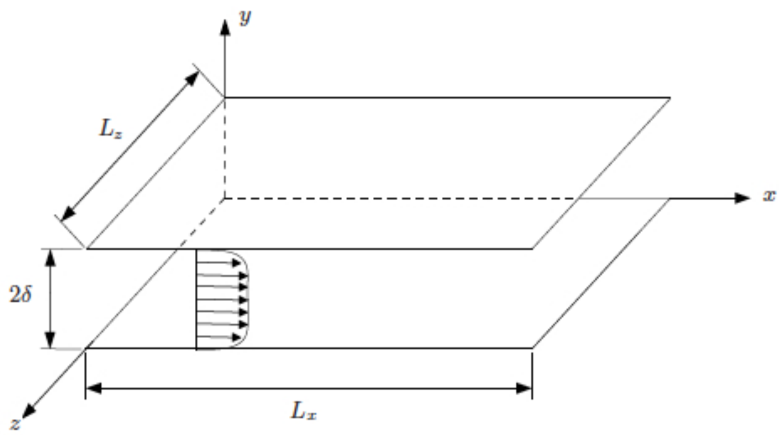

Figure1: Schematic of the channel flow.

2. Governing Equations

The governing equations for electrically conducting fluids in the presence of an externally imposed magnetic field in the limit of a low magnetic Reynolds number (simplified MHD equations) are [1]

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla p} + \frac{\mathbf{1}}{\boldsymbol{Re}}\boldsymbol{\nabla}^2\boldsymbol{u} + \boldsymbol{N}(-\boldsymbol{\nabla}\emptyset + \boldsymbol{u} \times \boldsymbol{B}_\circ) \times \boldsymbol{B}_\circ \quad (1)$$

$$\boldsymbol{\nabla}^2\emptyset = \boldsymbol{B}_\circ \cdot \boldsymbol{\omega} \quad (2)$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = \mathbf{0} \quad (3)$$

Where bold-face denotes vector quantities. In these equations, $\boldsymbol{u}$ is the velocity vector, p is the pressure, $\boldsymbol{B}_\circ$ is a unit vector pointing in the direction of the magnetic field, Re is the Reynolds number based on the channel half height, δ and the centerline velocity U_c, $\boldsymbol{\omega}$ is the vorticity vector and ∅ is the electric potential. The Stuart number is denoted by N, which is related to the Hartmann number through $Ha^2 = Re\ N$.

3. Numerical method and geometry

DNSs have been carried out using a pseudo-spectral Navier–Stokes solver, similar to the one employed in 0. The code uses Fourier representation in wall-parallel directions (x and z), using periodic boundary conditions, and Chebyshev representation in the wall-normal direction (y), using the Chebyshev–tau method. Aliasing errors are removed using the 3/2-rule. The time integration is carried out using a four-step third-order Runge–Kutta scheme for the nonlinear term and a second-order Crank–Nicholson scheme for the linear terms. The no-slip condition for the velocity is used at the walls. Simulations are carried out using a constant pressure constraint.

The schematic of the channel is shown in Figure 1. The channel dimensions in x, y and z directions are $2\pi\delta$, 2δ and $\pi\delta$, where δ is the channel half height. Simulations are performed with a constant bulk velocity in the x direction and the bulk Reynolds number is $Re_b = 2800$. The specifications of the numerical simulations are given in Table 1. The number of grid points in

Table 1: Specifications of the simulations. Direction of the magnetic field is denoted by $B_\circ$. The unit vectors in x, y and z directions are denoted by $\hat{e}_x$, $\hat{e}_y$ and $\hat{e}_z$.

and cases	$B_\circ$	Re_τ	Δx^+	Δy^+		Δz^+
				min	max	
B_0	-	180.0	8.84	0.054	4.4	4.42
B_x	$\hat{e}_x$	178.7	8.77	0.053	4.3	4.39
B_y	$\hat{e}_y$	175.1	8.60	0.053	4.3	4.30
B_z	$\hat{e}_z$	175.2	8.60	0.053	4.3	4.30

the x, y and z directions in all simulations are $N_x = 128$, $N_y = 129$ and $N_z = 128$. The Stuart number is $N = 0.01$.

4. Results and Discussion

The friction Reynolds number, given in Table 1, is reduced due to the presence of the magnetic field. It is observed that the magnetic field in the y and z directions result in a larger reduction of the wall shear, which is in agreement with previous studies [5].

Mean velocity profiles in wall units are presented in Figure 1. The results for case B_x are close to the plane channel flow, case B_0, whereas deviations are observed in the other cases with respect to case B_0. In case B_y the mean velocity has a shift after the buffer layer, in comparison with case B_0, while in case B_z the mean flow follows that of case B_0 in the inner layer and deviation occurs in the outer layer ($y^+ > 50$).

Reynolds stresses are given in Figure 2., in wall units. The streamwise component of the Reynolds stress in cases B_x and B_z follow those of case B_0 and only the peak value is slightly reduced due to the magnetic field. For case B_y significant differences are observed in comparison with case B_0. The major differences include a reduction of the peak close to the

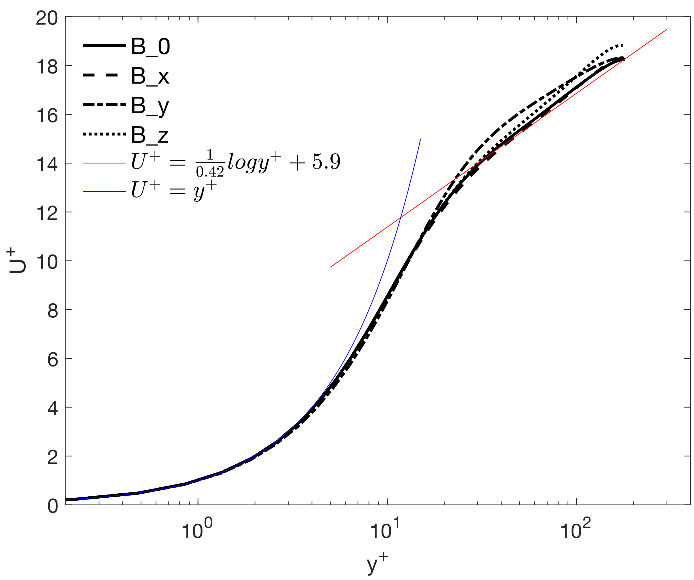

Figure 2: Streamwise mean velocity profiles in wall units. The velocity profiles are also compared to the logarithmic law of the wall.

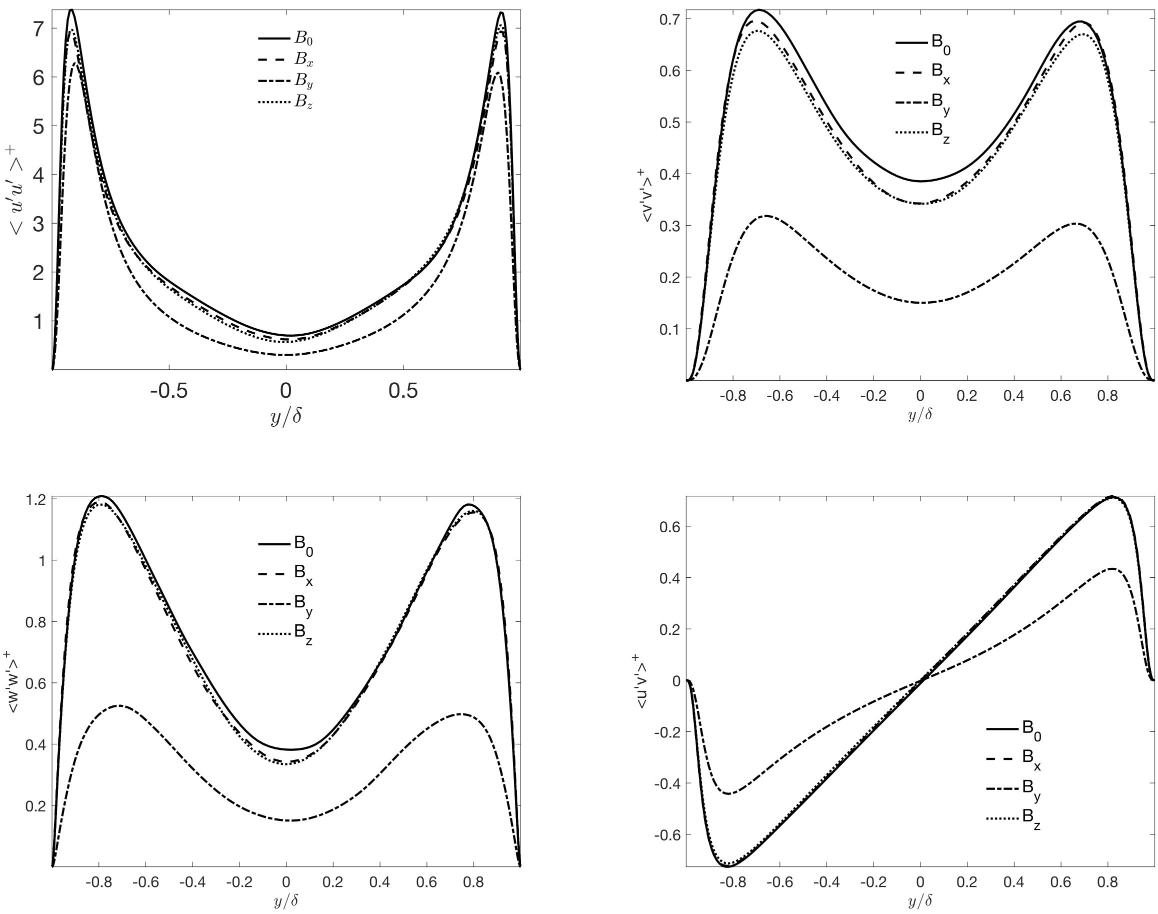

Figure 3: Mean Reynolds stress profiles in wall units across the channel.

wall and in the center of the channel. The spanwise and wall-normal components of the Reynolds stress tensor are almost the same in cases B_x, B_z and B_0 with only minor differences in the wall-normal component, whereas a significant reduction is observed in case B_y.

5. Conclusions

Direct numerical simulations (DNSs) were carried for a magnetohydrodynamic (MHD) turbulent channel flow at $Re_\tau = 180$. The effect of the magnetic field direction on the turbulent flow statistics was discussed. It was found that the magnetic field with the present Hartmann number has little effect on the mean velocity and Reynolds stresses, if it is applied in the streamwise and spanwise directions. Only minor differences were observed in these cases. When the magnetic field was applied in the wall-normal direction, a significant reduction in turbulence activity was observed. The mean velocity showed higher values outside the buffer layer, when presented in wall units. The Reynolds stresses were also significantly lower compared to the plane channel flow without a magnetic field. Further investigation of the flow field has been carried out, but the results are not presented here, for brevity. An analysis of the effect of the magnetic field direction on the flow structures and higher-order moments of the velocity will be presented at the conference.

Acknowledgements

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