Simultaneous Fault Detection and Control for Switched Linear Systems: An LMI Approach

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Abstract—This paper considers the problem of simultaneous fault detection and control (SFDC) in continuous-time switched linear systems. The problem is formulated as a multi-objective $H_{\infty}$ optimization problem and the results are presented in terms of linear matrix inequalities (LMI). Extended LMIs are used in the design of detector/controller to reduce conservatism. An average dwell-time approach is used to guarantee the stability and weighted $H_{\infty}$ performance of the closed loop system. Finally an illustrative example of SFDC design for the highly manoeuvrable technology vehicle is given to demonstrate the effectiveness of the approach.

Keywords—simultaneous fault detection and control, switched linear systems, linear matrix inequality, average dwell-time.

I. INTRODUCTION

The issue of model-based fault detection has attracted considerable attention during the last two decades and various methods have been developed for this purpose such as observer -based methods[1], fault detection filters [2], parity relations approach[3], parameter estimation approach[4] and so on. A survey of model-based fault detection methods can be found in [5]. Most of the mentioned model-based approaches use the open-loop model of the process and don’t consider the effect of controller in the design of fault detection module. As mentioned in [6] this may cause the faults to be hidden by control actions and as a result the generated residual is not sensitive enough to the faults or not robust enough against disturbances. This situation motivates us to investigate the problem of SFDC which unifies the control and disturbances. This situation motivates us to investigate the sensitivity of the faults or not robust enough against control actions and as a result the generated residual is not mentioned in [6] this may cause the faults to be hidden by controller in the design of fault detection module. As mentioned in [6] this may cause the faults to be hidden by control actions and as a result the generated residual is not sensitive enough to the faults or not robust enough against disturbances. This situation motivates us to investigate the problem of SFDC which unifies the control and disturbances.

In this paper the problem of simultaneous fault detection and control in continuous-time switched linear systems is investigated. Recently, the problem of fault detection in switched systems has become an interesting area for the researchers. But most of these works have used the open-loop model of the system and very few of them have considered the problem of SFDC for switched systems. In [14] the problem of SFDC for switched linear systems is formulated as non-convex inequality conditions and a two-step procedure is used to solve the problem. [15] has used a static observer to solve the problem of SFDC for both continuous-time and discrete-time switched linear systems which leads to equality constraints and conservative conditions. [16] has used a dynamic observer to tackle the problem which leads to strict LMI conditions but using dynamic observer imposes extra state variables to the dynamics of residual generator and increases complexity which is against the objectives of SFDC. In this paper we propose an $H_{\infty}$ formulation of SFDC problem by using a fault detection filter and extended linear matrix inequalities to reduce the conservativeness in the multi-objective optimization. The results are presented as strict LMIs without increasing the complexity of the augmented system.

Notation: Throughout this paper for real symmetric matrices $X$ and $Y$, $X > 0$ and $X < 0$ denote positive definiteness and negative definiteness and $X < Y$ means $X - Y$ is negative definite. For a matrix $A, A^T$ denotes it’s transpose. The Hermitian part of a square matrix $M$ is denoted by $\text{Her}(M): = M + M^T$. The symbol * in a matrix denotes the symmetric entries.

II. PROBLEM FORMULATION

Consider a switched linear system described by:

$$\begin{align*}
x(t) &= A_{\sigma(t)}x(t) + B_{1\sigma(t)}u(t) + B_{2\sigma(t)}d(t) + B_{3\sigma(t)}f(t) \\
y(t) &= C_{1\sigma(t)}x(t) + D_{11\sigma(t)}u(t) + D_{12\sigma(t)}d(t) + D_{13\sigma(t)}f(t)
\end{align*}$$

(1)
where \( x(t) \in \mathbb{R}^n \) is the system’s state, \( y(t) \in \mathbb{R}^m \) is the measured output, \( d(t) \in \mathbb{R}^p \) and \( f(t) \in \mathbb{R}^q \) represent the disturbance and fault input. The piecewise constant function \( \sigma(t) : [0, \infty) \to \mathbb{N} = \{1, 2, \ldots, N\} \) is a switching signal to specify the index of the active subsystem at each time instant, i.e., \( \sigma(t) = i \) means that the \( i \)’th subsystem is activated. The positive integer \( N \) denotes the number of subsystems. \( A_i, B_{i1}, B_{i2}, C_i, D_{i1}, D_{i2} \) are known matrices with appropriate dimensions.

The following filter is proposed for the dynamics of SFDC block:

\[
\begin{align*}
\dot{x}_c(t) &= A_{cc}(t)x_c(t) + B_{cc}(t)y(t) \\
u(t) &= C_{cc}(t)x_c(t) + D_{cc}(t)y(t) \\
r(t) &= C_{ct}(t)x_c(t) + D_{ct}(t)y(t)
\end{align*}
\]  

where \( x_c(t) \in \mathbb{R}^n \) is the filter’s state variable and \( r(t) \in \mathbb{R}^p \) is the residual signal. \( A_{cc}(t), B_{cc}(t), C_{cc}(t), D_{cc}(t) \) and \( A_{ct}(t), B_{ct}(t), C_{ct}(t), D_{ct}(t) \) are filter parameters to be designed later. Combining (1) and (2) we have the following augmented model:

\[
\begin{align*}
\dot{x}_{cl} &= \tilde{A}(t)x_{cl}(t) + \tilde{B}(d(t) + \tilde{B}_f f(t) \\
r(t) &= \tilde{C}(t)x_{cl}(t) + \tilde{D}(d(t) + \tilde{D}_f f(t) \\
z(t) &= [x(t)^T x_c(t)^T]^T
\end{align*}
\]

where \( x_{cl} = [x(t)^T \ x_c(t)^T]^T \) and :

\[
\tilde{A}(t) = \begin{bmatrix} A(t) + B(t)D(t) \ B(t)C(t) \\ A(t) \end{bmatrix} \\
\tilde{B}(d) = \begin{bmatrix} B(t)D(t) \ B(t) \end{bmatrix} \\
\tilde{C}(t) = \begin{bmatrix} C(t)D(t) \ C(t) \end{bmatrix} \\
\tilde{D}(d) = \begin{bmatrix} D(t) \ D(t) \end{bmatrix}
\]

The simultaneous fault detection and control problem to be addressed in this paper can be expressed as follows: Given the plant model (1), design the SFDC block such that the augmented model (3) is stable and at the same time meets some control objectives (decreasing the effect of fault and disturbance on regulated output) and detection objectives (decreasing the effect of disturbance on residual and increasing the effect of fault on it). These objectives can be mathematically expressed as the following weighted \( H_{\infty} \) performances.

\[
\int_0^\infty z(t)^Tz(t)dt < \gamma_1^2 \int_0^\infty e^{-\eta_1 t}d(t)^Td(t)dt
\]

\[
\int_0^\infty z(t)^Tz(t)dt < \gamma_2^2 \int_0^\infty e^{-\eta_2 t}f(t)^Tf(t)dt
\]

\[
r^T(t)r(t) < \gamma_3^2 \int_0^\infty e^{-\eta_3 t}d(t)^Td(t)dt
\]

\[
r^T(t)r(t) < \gamma_4^2 \int_0^\infty e^{-\eta_4 t}f(t)^Tf(t)dt
\]

in which (5), (6) are control objectives and (7),(8) are detection objectives. \( \eta_1, \gamma_1 \) are positive constants and \( r_e \) is defined as follows

\[
r_e = r - r_F
\]

Thus (8) means that the residual signal should robustly track a filtered version of fault signal. The above performance indexes should be satisfied simultaneously. As mentioned in [17] introducing slack variables such that the Lyapunov matrices are decoupled from system matrices, reduces conservatism in multi-objective problems and improves the results of LMI feasibility conditions. Therefore the conservativeness reduction method in [16] is used in theorems 1-4 to satisfy the above performances.

**Definition 1** [16]: For each switching signal \( \sigma \) and each \( t \geq \tau \geq 0 \) let \( N_{\sigma}(t, \tau) \) denote the number of discontinuities of \( \sigma \) in the open interval \( (t, \tau) \). For given \( \tau_0, N_0 \) if \( N_{\sigma}(t, \tau) \leq N_0 + (\tau - \tau)/\tau_a \) then \( \tau_a \) is called the average dwell-time (ADT).

**III. MAIN RESULTS**

In the following theorem a sufficient condition is given to satisfy control objective (5).

**Theorem 1** : Consider the augmented switched linear system (3) with \( f(t) = 0 \) and given constants \( \alpha > 0, \lambda > 0, \mu \geq 1 \) and \( \gamma_1 > 0 \). For any switching signal with average dwell-time satisfying (11), the corresponding system is exponentially stable and satisfies (5) for some \( \gamma_1 \), if there exist symmetric matrices \( P_{ij} > 0 \) and matrices \( X_i, Y_i, Q_i, L_i, M_p, S_i, D_c \) such that the following LMIs hold.

\[
\tau > \tau^* = \frac{\ln \mu}{\alpha}
\]

\[
[aP_{11} + E_{11} \ P_{11} + E_{12} \ E_{13} \ E_{14}] < 0 \quad \forall j \in \mathbb{N}
\]

\[
P_{11} < \mu P_{11} \quad \forall i, j \in \mathbb{N}, i \neq j
\]

where

\[
E_{11} = \begin{bmatrix} H(e(A_iX_j + B_1L_i)) & A_i + B_1D_cC_i \\ Y_jA_j + M_jC_i + (Y_jA_j + M_jC_i)^T \end{bmatrix}
\]
(14) Define $P_{ij} = T_j^T P_i T_j$. By replacing $B_{2dj}^T, C_{2j}^T, B_{dj}$, $A_j^T$ from (4) in (19) and using the structure introduced in (18) we obtain

$$T_j^T \tilde{A}_j F_T j = \begin{bmatrix} A_j \bar{X}_j + B_j \bar{L}_j & A_j \bar{B}_j \bar{C}_j \bar{C}_j \bar{I} \\ Y_j \bar{A}_j + M \bar{C}_j \end{bmatrix} \quad (20)$$

where

$$Q_j = Y_j \bar{A}_j \bar{X}_j + V_j \bar{A}_j \bar{U}_j + V_j \bar{B}_j \bar{C}_j \bar{X}_j + Y_j \bar{B}_j \bar{C}_j \bar{U}_j$$

and

$$C_{2j}^T F_T j = \begin{bmatrix} C_{2j} \bar{X}_j + D_{23j} \bar{L}_j & C_{2j} + D_{23j} \bar{B}_j \bar{C}_j \bar{I} \end{bmatrix} \quad (21)$$

and also

$$T_j^T \tilde{B}_{dj} = \begin{bmatrix} B_{2j} + B_{2j} \bar{D}_{cj} D_{11j} \\ Y_j \bar{B}_{dj} + M \bar{D}_{11j} \end{bmatrix} \quad (22)$$

$$T_j^T P_{1j} T_j = \bar{P}_{ij} \quad (23)$$

$$T_j^T F_T j = \begin{bmatrix} X_j & Y_j \end{bmatrix}^T \quad (24)$$

where

$$S_j = Y_j X_j + V_j U_j \quad (25)$$

By using these new variables (19) turns to (12). Based on (12):

$$\begin{bmatrix} X_j + X_j^T & I + S_j \\ I + S_j & Y_j + Y_j^T \end{bmatrix} > 0 \quad (27)$$

From (27) we conclude

$$X_j + X_j^T > 0 \quad , \quad Y_j + Y_j^T > 0$$

and as a result $X_j$ and $Y_j$ are non-singular. Using a congruence transformation on (27) with $[X_j^T \quad -I]^T$ we obtain

$$X_j^T - S_j X_j^{-1} - X_j^T - X_j^T S_j + Y_j + Y_j^T > 0$$

so:

$$(-S_j + Y_j X_j) X_j^{-1} + X_j^T (-S_j + X_j^T Y_j) > 0$$

This means that $S_j - Y_j X_j$ is non-singular and matrices $U_j$ and $V_j$ exist such that

$$V_j U_j = S_j - Y_j X_j$$

(31)

Determining matrices $U_j$ and $V_j$ the parameters of SFDC block are obtained as follows.

$$D_{cj} = D_{cj}$$

$$C_{cj} = (L_j - D_{cj} C_j \bar{X}_j) U_j^{-1}$$

$$B_{cj} = V_j^{-1}(M_j - Y_j B_{cj} D_{cj})$$

$$A_{cj} = V_j^{-1}(Q_j - Y_j A_j \bar{X}_j - V_j B_j C_j \bar{X}_j - X_j B_{cj} C_j \bar{U}_j - Y_j B_{cj} D_{cj} U_j)^{-1}$$

(32)

This ends the proof.

The next theorem provides a sufficient condition for detection objective (6):

**Theorem 2:** Consider the augmented switched linear system (3) with $d(t) = 0$ and given constants $\alpha > 0$, $\lambda > 0$.
Given $\mu \geq 1$ and $\gamma_3 > 0$. For any switching signal with ADT satisfying (11) the corresponding system is exponentially stable and satisfies (6) for some $\eta_2$, if there exist symmetric matrices $P_{2j} > 0$ and matrices $X_j, Y_j, Q_j, E_j, M_j, S_j, D_{Cj}$ such that the following LMIs hold.

$$
\begin{bmatrix}
\alpha P_{2j} + E_{11} & P_{3j} + E_{12} & E_{13} & E_{14} \\
* & E_{22} & E_{23} & 0 \\
* & * & -\gamma_2 I & E_{24} \\
* & * & * & -I
\end{bmatrix} < 0 \quad (33)
$$

where

$$
E_{11} = \begin{bmatrix} B_1 D_{Cj} D_{12j} + B_{3j} \\
Y_j B_{3j} + M_j D_{22j} \\
C_1 \gamma_2 I \\
\end{bmatrix} E_{14} = \begin{bmatrix} B_2 D_{Cj} D_{12j} + B_{3j} \\
Y_j B_{3j} + M_j D_{22j} \\
C_2 \gamma_2 I \\
\end{bmatrix}
$$

and given constants $\alpha > 0, \lambda > 0, \mu \geq 1$ and $\gamma_3 > 0$. For any switching signal with average dwell-time satisfying (11) the corresponding system is exponentially stable and satisfies (8) for some $\eta_4$, if there exist symmetric matrices $P_{4j} > 0$ and matrices $X_j, Y_j, Q_j, E_j, M_j, S_j, D_{Cj}$ such that the following LMIs hold.

$$
\begin{bmatrix}
\alpha P_{4j} + E_{11} & P_{4j} + E_{12} & E_{13} & E_{14} \\
* & E_{22} & E_{23} & 0 \\
* & * & -\gamma_2 I & E_{24} \\
* & * & * & -I
\end{bmatrix} < 0 \quad \forall j \in \mathbb{N} \quad (39)
$$

Proof: The proof is similar to the proof of theorem 1 and is omitted here because of space limitation.

The next theorem provides LMIs for control objective (7).

**Theorem 3:** Consider the augmented switched linear system (3) with $f(t) = 0$ and given constants $\alpha > 0, \lambda > 0, \mu \geq 1$ and $\gamma_3 > 0$. For any switching signal with average dwell-time satisfying (11), the corresponding system is exponentially stable and satisfies (7) for some $\eta_3$, if there exist symmetric matrices $P_{3j} > 0$ and matrices $X_j, Y_j, Q_j, L_j, M_j, S_j, D_{Cj}$ such that the following LMI’s hold.

$$
\begin{bmatrix}
\alpha P_{3j} + E_{11} & P_{3j} + E_{12} & E_{13} & E_{14} \\
* & E_{22} & E_{23} & 0 \\
* & * & -\gamma_2 I & E_{24} \\
* & * & * & -I
\end{bmatrix} < 0 \quad (36)
$$

where

$$
E_{11} = \begin{bmatrix} B_1 D_{Cj} D_{12j} + B_{3j} \\
Y_j B_{3j} + M_j D_{22j} \\
C_1 \gamma_2 I \\
\end{bmatrix} E_{14} = \begin{bmatrix} B_2 D_{Cj} D_{12j} + B_{3j} \\
Y_j B_{3j} + M_j D_{22j} \\
C_2 \gamma_2 I \\
\end{bmatrix}
$$

Proof: Define $\zeta = [x_T y_T]$. Combining (4) and (10) dynamics of closed-loop system with $d(t) = 0$ is as follows.

$$
\zeta(t) = \tilde{A}_i \zeta(t) + \tilde{B}_i f(t) \\
r_\eta(t) = \tilde{C}_i \zeta(t) + \tilde{D}_i f(t) \quad (42)
$$

where

$$
\tilde{A}_i = \begin{bmatrix} A_F & 0 & 0 & 0 \\
0 & A_1 + B_1 D_{Cj} C_{1l} & B_1 C_{1l} & 0 \\
0 & B_2 C_{1l} & A_{1l} & 0 \\
B_1 & B_2 C_{1l} & 0 & 0
\end{bmatrix}
$$

$$
\tilde{B}_i = \begin{bmatrix} B_1 D_{Cj} D_{12i} + B_{3i} \\
Y_j B_{3j} + M_j D_{22i} \\
C_1 \gamma_2 I \\
D_{rj} C_{1l} & C_{rl} & \tilde{D}_j = D_r D_{12j} - D_F \quad (43)
\end{bmatrix}
$$

$$
\tilde{C}_i = [C_F D_{rj} C_{1l} C_{rl}]
$$

Proof: The proof is similar to the proof of theorem 1 and is omitted here because of space limitation.
Following the same procedure in the proof of theorem 1 (39) and (40) are obtained.

Algorithm: Given the constants \( \alpha > 0, \lambda > 0, \mu \geq 1 \) a feasible solution to SFDC problem is obtained by the following optimization problem.

\[
\min a(y_1 + y_2) + b(y_3 + y_4)
\]

subject to (12), (13), (33), (34), (36), (37), (39), (40)

(44)

where \( a \) and \( b \) are positive constants determining the importance of detection objectives and control objectives with respect to each other. In the case that control objectives are more important than detection objectives constant \( a \) is chosen bigger compared with \( b \). Otherwise if high priority is given to detection objectives then \( b \) is chosen bigger than \( a \).

After the parameters of the filter are determined, the residual evaluation function \( J_r(t) \) and the threshold \( J_{th} \) can be selected as

\[
J_r(t) = \frac{1}{T} \int_{t-T}^{t} r(s) \, ds
\]

(45)

\[
J_{th} = \sup_{d(t) \in D, J_r(t) > 0} J_r(t)
\]

(46)

where \( T \) denotes the evaluation time step. Finally the occurrence of fault can be detected by the following logic rule.

\[
\begin{align*}
J_r(t) & \leq J_{th} \quad \text{no alarm} \\
J_r(t) & \geq J_{th} \quad \text{alarm}
\end{align*}
\]

(47)

Remark 1: The main advantage of the method introduced here to one presented in [16] is that [16] uses a correction signal to eliminate nonlinearities in the matrix inequalities which imposes some dynamics in addition to the dynamics of the observer itself and this makes an SFDC block with too many state variables and increases complexity which is in contrast with the aim of simultaneous fault detection and control. But the method presented in this paper uses only a filter with the same dimension as the static observer in [16] without imposing extra state variables.

IV. Simulation

In this section a HiMAT vehicle given in [11] is used to show the effectiveness of the introduced method. As shown in TABLE I the switched system consists of two operating points in the flight envelope. Therefore the switched system includes two subsystems and two state variables \( x = [\delta \ z] \), where \( \delta \) and \( z \) denote angle of attack and pitch rate respectively. The parameters of the switched system are defined in TABLE II.

Choose \( \alpha = 0.06, \lambda = 0.1, \mu = 1.1 \). According to (11) \( \tau^*_2 = 1.59 \) so we choose a switching signal with \( \tau = 2 \) (Fig. 1). The parameters of the low-pass filter in (11) are chosen as follows.

\[
\begin{bmatrix}
A_r & B_r \\
C_r & D_r
\end{bmatrix} =
\begin{bmatrix}
-3 & 0 & 1 & 1 \\
0 & -5 & 1 & 1 \\
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0
\end{bmatrix}
\]

A band-limited white noise with the power of 0.01 is used as the disturbance input and the fault signal is \( f(t) = [0.2 \ 0]^T \) which occurs at \( t = 50 \) s.

For the purpose of designing SFDC block parameters with more emphasis of detection, the parameters of algorithm 1 are chosen as \( a = 0.0001, b = 1 \) and the LMI’s are solved. This leads to \( y_1 = 0.4, y_2 = 0.5, y_3 = 0.15, y_4 = 1.01 \). In the second scenario more emphasis is put on control objectives by choosing \( a = 0.01, b = 1 \). In this case \( y_1 = 0.01, y_2 = 0.01, y_3 = 0.33, y_4 = 1.25 \) which shows better performance in control and worse performance in detection. SFDC block parameters for these two scenarios are shown in TABLE III and TABLE IV. Fig. 2 shows the evaluation function of residuals generated by the two blocks according to (45) with \( T = 20s \). The thresholds are obtained as 0.0059 and 0.00037 respectively. It can be seen that the first block is better in fault detection and the fault is detected in 1.4 seconds while the second block detects it in 15.7 seconds. Fig. 3 shows the regulated output for the designed SFDC blocks. It can be seen that the block designed with emphasis on control shows a better performance in this case.

### TABLE I. Two operating points of Himat vehicle

<table>
<thead>
<tr>
<th>Operating point</th>
<th>Mach</th>
<th>Altitude</th>
<th>Angle of attack</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6</td>
<td>20000 ft</td>
<td>1.48 deg</td>
</tr>
<tr>
<td>2</td>
<td>0.9</td>
<td>25000 ft</td>
<td>4.27 deg</td>
</tr>
</tbody>
</table>

### TABLE II. Parameters of the switched system

<table>
<thead>
<tr>
<th>First subsystem</th>
<th>Second subsystem</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 = [-1.35 \ 0.98] )</td>
<td>( A_2 = [-1.87 \ 0.98] )</td>
</tr>
<tr>
<td>( B_{11} = B_{21} = B_{31} = )</td>
<td>( B_{12} = B_{22} = B_{32} = )</td>
</tr>
<tr>
<td>( [-0.13 \ -0.013] )</td>
<td>( [ -0.16 \ -0.005 ] )</td>
</tr>
<tr>
<td>( [-14.2 \ 10.75] )</td>
<td>( [-29.2 \ 21.3] )</td>
</tr>
<tr>
<td>( C_{11} = C_{21} = [1 \ 0] )</td>
<td>( C_{21} = C_{22} = [1 \ 0] )</td>
</tr>
<tr>
<td>( D_{111} = D_{121} = D_{211} = D_{221} = )</td>
<td>( D_{112} = D_{122} = D_{212} = D_{222} = )</td>
</tr>
<tr>
<td>( [0 \ 0] )</td>
<td>( [0 \ 0] )</td>
</tr>
</tbody>
</table>

### TABLE III. Parameters of SFDC block for the first system

<table>
<thead>
<tr>
<th>Emphasis on detection</th>
<th>Emphasis on control</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{c1} = [-27.22 \ -52.35] )</td>
<td>( [10^3 \ 0.560 \ -157] )</td>
</tr>
<tr>
<td>( B_{c1} = [-25.11 \ -0.93] )</td>
<td>( [-26.23 \ -0.48] )</td>
</tr>
<tr>
<td>( C_{c1} = [0.03 \ -4.62] )</td>
<td>( [10^4 \ 0.027 \ -4.28] )</td>
</tr>
<tr>
<td>( D_{c1} = [170.49 \ 219.55] )</td>
<td>( [301.36 \ 380.19] )</td>
</tr>
</tbody>
</table>

TABLE I. Two operating points of Himat vehicle

TABLE II. Parameters of the switched system

TABLE III. Parameters of SFDC block for the first system
**TABLE IV. PARAMETERS OF SFDC BLOCK FOR THE SECOND SYSTEM**

<table>
<thead>
<tr>
<th>Emphasis on detection</th>
<th>Emphasis on control</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{c2} )</td>
<td>( -26.53 )</td>
</tr>
<tr>
<td></td>
<td>(-0.40)</td>
</tr>
<tr>
<td></td>
<td>( 10^3)</td>
</tr>
<tr>
<td></td>
<td>(-1.69)</td>
</tr>
<tr>
<td>( B_{c2} )</td>
<td>(-26.61)</td>
</tr>
<tr>
<td>( C_{c2} )</td>
<td>( 0.019 )</td>
</tr>
<tr>
<td></td>
<td>( 0.024 )</td>
</tr>
<tr>
<td></td>
<td>( 10^4)</td>
</tr>
<tr>
<td></td>
<td>( -3.65)</td>
</tr>
<tr>
<td></td>
<td>(-4.78)</td>
</tr>
<tr>
<td>( D_{c2} )</td>
<td>( 142.28)</td>
</tr>
<tr>
<td></td>
<td>( 188.92)</td>
</tr>
</tbody>
</table>

**Figure 1.** Switching signal

**Figure 2.** Evaluation functions for residuals generated with emphasis on control and detection

**Figure 3.** Regulated output

**V. CONCLUSION**

This paper considers the problem of SFDC for switched linear systems. The SFDC objectives are formulated as \( H_\infty \) optimization problem in terms of LMI’s. An extended LMI formulation is used obtain less conservative results in the multi-objective optimization. Using some congruence transformations and variable changes the results are presented in strict LMI’s. Finally the approach is applied to the longitudinal motion of a HiMAT vehicle to show the effectiveness of the method and it’s shown that there is a trade-off between fault detection and control objectives.

**REFERENCES**


