Implementation and Performance Comparison of Indirect Kalman Filtering Approaches for AUV Integrated Navigation System using Low Cost IMU

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Abstract: Strapdown Inertial Navigation System (SINS) estimates position, velocity, and attitude of vehicle using the signals measured by accelerometer and gyroscope and is based on dead-reckoning principle. Due to different imperfections in measurements, and the consecutive integration of the acceleration signals, estimation error increases with time and it is acceptable only for short times in the low cost SINS. In order to reduce the error, auxiliary sensors together with inertial sensors are utilized and to combine the data estimated by SINS with the signals measured by auxiliary sensors, data fusion methods based on direct and indirect Kalman filtering is used. The feedforward and feedback structures are two common approaches of the indirect filtering. In this paper, an underwater integrated navigation system has been designed using indirect filtering approaches due to lower computational load and higher reliability with respect to the direct approach. The auxiliary sensors used consist of DVL, Gyrocompass, and depthmeter. The performance of the designed system has been studied using real measurements. The experimental results showed that the root mean square error in the estimated position for feedforward structure is reduced from 3.2 to 0.2 percent of the travelled distance when using feedback structure.

Keywords: Strapdown inertial Navigation System, Integrated navigation system, Indirect Filtering approaches, Feedforward and Feedback Kalman filter

1. Introduction

There is increasing demand for small-sized and low-cost Strapdown inertial navigation systems (SINSs) for use in many applications such as land vehicle navigation, unmanned vehicles (underwater and aerial) and general aviation. In the SINS, the principle of the dead-reckoning is used to calculate the vehicle’s position. SINSs consist of three orthogonal accelerometers and three orthogonal gyroscopes which uses for estimating of the position, velocity, and orientation of the vehicle. In the low cost SINS mainly due to different noise sources in accelerometers and gyroscopes, and the successive integration of the acceleration, the position error increases with time and the growth of the error is moderate for short periods of time [1]. In order to bound the error growth, the SINSs are used together with other navigation aids. In underwater integrated navigation systems, it is common to use the auxiliary sensors such as Doppler Velocity Log (DVL), orientation sensor, pressure sensor, global positioning system (GPS) to reduce the position error [2]. Unfortunately, the GPS signals are not receivable under the water. For this reason, the INS/GPS integrated systems for underwater navigation are limited to shallow-water applications where the vehicle must come to the surface regularly to correct its position [3, 4]. In order to combine the SINS output with the auxiliary measurements, Kalman filtering techniques are often used.

The Kalman filter is implemented in two approaches, direct and indirect filtering. Direct filtering is rarely used due to its high computational load and low reliability. The indirect filtering is utilized in mainly all integrated inertial navigation systems. The two principle structures of the indirect filtering are the feedforward and the feedback Kalman filter. In this paper, an integrated inertial navigation system has been implemented using indirect Kalman filtering approaches for underwater application. Auxiliary sensors used in the designed system include DVL, orientation sensor, and depthmeter. Unlike the GPS, using these sensors causes the integrated navigation system to navigate independently utilizing sensors attached to vehicle.

Until now, the performance comparison of the feedforward and the feedback structures has been rarely done in the literatures. For example, Ragel and Farooq [5] discussed the possibility of using the forward and the feedback Kalman filter for an aided INS in the AUV application using simulated data. Hence, in this paper, the performance of two different structures of the indirect filtering approach is investigated using real data obtained from the inertial sensors and the auxiliary sensors consisting of the DVL, Gyrocompass, and depthmeter.

The structure of this paper is organized as follows. After introduction, implementation approaches of the Kalman filter are reviewed in section 2. In section 3, the equations related to the implementation of the indirect filtering approaches are derived. In section 4, the performance of the feedforward and feedback structures for designed integrated navigation system is compared using the results of the experimental tests. Finally, conclusions are presented in section 6.
2. Kalman filter implementation

The Kalman filter is a very effective and powerful method for incorporating noisy outputs of the multiple sensors to estimate the state of a system with uncertain dynamics. In integrated inertial navigation systems, the Kalman filter has been implemented in two approaches, total state space and error state space methods which, respectively named as direct and indirect filtering in the literature associated with navigation [6].

In the direct filtering, the output of the inertial sensors and the auxiliary measurements (velocity, orientation, and depth) are applied to filter and the Kalman filter incorporates the inertial data with auxiliary measurements [7] (Fig.1).

In this Approach has some major drawbacks, such as the Kalman filter is located in the SINS loop, therefore if the Kalman filter fails, the whole system will not work and so filter equations are executed with high sampling frequency of the inertial sensors in the iteration loop. Due to the intensive computations in the Kalman filter, the incorporation rate of the inertial data with auxiliary measurements is severely limited [8].

Indirect filtering in compare to direct filtering had some advantage, such as the state space model can be based on a linearized system model [8], moreover, the Kalman filter is located outside of the SINS loop, thus if an error occurs in the filter, the system will continue to work and the position of the vehicle will estimate by the SINS and also the iteration rate of the filter significantly reduced [6]. Therefore, the indirect filtering can be utilized in all integrated inertial navigation systems.

The indirect Kalman filter is also implemented in two structures, feedforward and feedback. In the feedforward structure, as shown in Fig.2, the SINS output and Auxiliary measurements are compared each other and the Kalman filter estimates the errors of the position, velocity and orientation. The optimum estimates of the position, velocity and orientation are obtained by deducting the estimated errors from the SINS output.

In this structure, the SINS acts independently, that is, it has no knowledge of the existence of the Kalman filter or the auxiliary sensors. Thus, in the event that the Kalman filter doesn’t perform correctly or the auxiliary measurements are not available, the uncorrected SINS outputs will be achieved.

In addition, the SINS drift is increased unboundedly [6]. If the errors of the inertial system remain small, the linear dynamics model in the Kalman filter is acceptable. But if the low cost inertial sensors are used in the system, the estimated errors by the Kalman filter will drift with time [9]. The feedback structure removes the deficiency of the feedforward structure (Fig.3).

3. Implementation of integrated inertial navigation system

3.1 Feedforward structure

According to the Fig.3, in the feedback structure similar to the feedforward structure, the inertial system’s errors are estimated in the Kalman filter, but the estimated state is feedback into the SINS to correct it. Therefore, the magnitude of the inertial system’s errors remains small. Furthermore, the SINS drift is limited. For this reason, when the filter executes improperly or the auxiliary sensors are inaccessible, the corrected SINS output will be available [6].
direction of the north, east, and local vertical. The orientation of the vehicle is described by the roll, pitch and heading angles.

3.1.1 The SINS

Calculation of the velocity, depth, and orientation of the vehicle is carried out in the SINS. To calculate the velocity, first, it is necessary to transform the acceleration signals measured by IMU from the body frame to the navigation frame.

Afterwards, the transformed signals are integrated to obtain the vehicle velocity in the navigation frame. It is noted that the effects of the gravitational acceleration and additional forces caused by the Earth rotation must be corrected before integration.

The equations related to the calculation of the velocity can be expressed as follows [1]:

\[
\begin{align*}
\mathbf{f}_n^* &= \mathbf{C}_b^m \mathbf{f}_b^m \\
\mathbf{f}_e^n &= \mathbf{f}_n^* - (2\mathbf{\omega}_{me}^n + \mathbf{\omega}_{en}^n) \times \mathbf{V}_e^n + \mathbf{g}_f^n \\
\mathbf{V}_{D,SINS}^n &= \int \mathbf{f}_e^n dt 
\end{align*}
\]

(1) (2) (3)

To calculate the vehicle depth, the velocity in vertical direction calculated by Eq. (3), is integrated:

\[
\mathbf{d}_{SINS} = \int \mathbf{V}_{D,SINS} dt 
\]

(4)

The orientation of the vehicle relative to the navigation frame is computed by the following equations [1]:

\[
\begin{align*}
\mathbf{\phi}_{SINS} &= \int (\mathbf{\omega}_3 \sin \phi + \mathbf{\omega}_2 \cos \phi \tan \theta + \mathbf{\omega}_4) dt \\
\mathbf{\theta}_{SINS} &= \int (\mathbf{\omega}_3 \cos \phi - \mathbf{\omega}_2 \sin \phi) dt \\
\mathbf{\psi}_{SINS} &= \int (\mathbf{\omega}_3 \sin \phi + \mathbf{\omega}_2 \cos \phi \tan \theta) dt 
\end{align*}
\]

(5) (6) (7)

3.1.2 SINS error equations

In this section, the equations related to the errors of the velocity, orientation, and depth in a SINS are briefly reviewed. The proof of these equations has been completely presented in [1].

The linearized equations related to the velocity and orientation errors may be expressed as follows:

\[
\begin{align*}
\delta \mathbf{V}^n &= \left[ \mathbf{f}^* \times \right] \mathbf{\Psi} + \mathbf{C}_b^m \delta \mathbf{\alpha} - \left( 2 \mathbf{\omega}_{me}^n + \mathbf{\omega}_{en}^n \right) \times \delta \mathbf{V}^n \\
&\quad - \left[ 2 \mathbf{\delta \omega}_{me}^n + \mathbf{\delta \omega}_{en}^n \right] \times \delta \mathbf{V}^n \\
\delta \mathbf{\Psi} &= - \mathbf{\omega}_{me}^n \times \delta \mathbf{\Psi} + \mathbf{\omega}_{en}^n - \mathbf{C}_b^m \mathbf{\delta \omega}_{en}^m \\
&\quad \mathbf{\Psi} = \left[ \delta \phi, \delta \theta, \delta \psi \right] 
\end{align*}
\]

(8) (9)

Where \( \mathbf{\Psi} \) is the orientation error vector. \( \delta \phi \) and \( \delta \theta \) are the tilt errors and \( \delta \theta \) is the heading error. On the condition that small angle approximations are valid, these errors may be assumed equal to the error of the Euler angles, i.e. the roll, pitch, and yaw errors. \( \delta \phi \) and \( \delta \theta \) indicate the errors in the signals measured by accelerometer and gyroscope.

These errors are modelled in the Kalman filter as additive Gaussian white noise. \( \delta \mathbf{\omega}_{en}^m \) is the error in the angular rate of the navigation frame with respect to the inertial frame. Eventually, the depth error may be expressed by the following equation:

\[
\delta \mathbf{d} = \delta \mathbf{N}_D
\]

(10)

3.1.3 Dynamic model

Many dynamical systems are modeled by Continuous time state space model. By combining Equations (8), (9)
and (10), continuous state space model in the indirect approach may be expressed as follows:
\[ \dot{x} = F \dot{x} + L w \]  
(11)

Where \( \dot{x} \) is the error state vector of the system may be included the position, velocity, orientation, \( F \) is the system error matrix and is derived from the Eqs. (8), (9), and (10). \( L \) is a dynamic noise distribution matrix, random process \( w \) is a white-noise vector with zero mean and a power spectral density \( Q \).

The error state vector of the system consists of the errors of the depth, velocity, and orientation and may be expressed as follows:
\[ \dot{x} = [\phi/4, \phi, \phi, \dot{\phi}, \dot{\phi}, \dot{\phi}]^T \]  
(12)

In order to implement the Kalman filter, it is necessary to express the system error Eq. (11) in discrete form as follows:
\[ \dot{x}_{k+1} = A_k \dot{x}_k + q_k \]  
(13)

Where \( \dot{x}_k \) and \( \dot{x}_{k+1} \) are the system error states at times \( t_k \) and \( t_{k+1} \), respectively. \( q_{k+1} \) is zero mean Gaussian white-noise process with known covariance \( Q_{k+1} \), \( A_k \) is the state transition matrix. In order to compute the matrix \( A_k \), the following equation is used:
\[ A_k = \exp(F \Delta t) \]  
(14)

Matrix \( Q_k \) may be computed using Trapezoidal integration is given by [9]:
\[ Q_k = \frac{1}{2} (A_k \hat{L} Q_k \hat{L}^T A_k^T + L Q_k L^T) \Delta t \]  
(15)

Where \( \Delta t = t_{k+1} - t_k \) is the time step of the discretization.

3.1.4 Measurement model

As shown in Fig.4, the difference between the data estimated by SINS and the signals measured by auxiliary sensors are calculated to obtain the filter innovations \( \dot{z} \) which is applied to the Kalman filter as the measurements. The relationship between the filter innovations and the error states is linear and may be expressed as follows:
\[ \dot{z} = \begin{bmatrix} d_{SINS}(t_k) - d_m(t_k) \\ V_{SINS}^n(t_k) - V_m^n(t_k) \\ \Theta_{SINS}(t_k) - \Theta_m(t_k) \\ \Psi(t_k) \end{bmatrix}, \dot{x}_k = \begin{bmatrix} \dot{d}(t_k) \\ \dot{V}^n(t_k) \\ \dot{\Theta}(t_k) \\ \dot{\Psi}(t_k) \end{bmatrix} \]  
(16)

The filter innovations at time \( t_k \) may be expressed in terms of the error states by the following matrix form:
\[ \dot{z}_{k+1} = H_k \dot{x}_k + r_k \]  
(17)

Where \( H_k \) is the measurement model matrix and is expressed as follows:
\[ H_k = [I_{5 \times 7}] \]  
(18)

Since in the indirect approach, the nonlinear state space model is converted to a linear model, the kalman filter used in this approach is also linear whose prediction and correction stages is followed by set of equations.

3.1.5 The Kalman filter

Equations (13) and (17) are the system and measurement equations needed to construct a Kalman filter. Kalman filter is comprised of two stages. The prediction step, where the next state of the system is predicted using the dynamic model and the state estimated from the previous step, and the correction stage, where the current state of the system is estimated based on new measurements. These two stages are implemented by the following equations [1]:

**Prediction stage:**
\[ \hat{x}_{k,:} = A_{k-1} \hat{x}_{k-1,:} \]  
(19)
\[ P_k = A_{k-1} P_{k-1} A_{k-1}^T + Q_{k-1} \]  
(20)

**Correction stage:**
\[ K_k = P_k H_k^T \left( H_k P_k H_k^T + R_k \right)^{-1} \]  
(21)
\[ \hat{x}_{k,:} = \hat{x}_{k,:} + K_k (z_k - H_k \hat{x}_{k,:}) \]  
(22)
\[ P_k = P_k - K_k \left( H_k P_k H_k^T + R_k \right) K_k^T \]  
(23)

Where \( \hat{x}_{k,:} \) and \( P_k \) are the predicted state and estimation covariance, respectively, \( \hat{x}_{k,:} \) and \( P_k \) are the corrected state and estimation covariance, respectively, \( K_k \) is the Kalman gain which determines how much the predicted values are corrected at time step \( t_k \).

3.1.6 SINS correction

The data estimated by the SINS are corrected after each correction stage using the errors estimated in that stage. The equation related to the SINS correction may be expressed as follows:
\[ x_c(t_k) = x_{SINS}(t_k) - \hat{m} \]  
(24)

Where \( x_{SINS} \) is the SINS state and consists of the vehicle depth, velocity, and orientation. \( \hat{m} \) is the estimated errors in the correction stage of the filter and \( x_c \) is the corrected state of the SINS. Vehicle position in the horizontal directions (latitude and longitude) is calculated by integrating the velocity corrected using Eq. (24) which is given by:
\[ \dot{L}_c = \frac{V_{N,c}}{R_0 + d_c} \]  
(25)
\[ \dot{d}_c = \frac{V_{E,c}}{R_0 + d_c} \]  
(26)

3.2 Feedback structure

The block diagram of the integrated navigation system implemented in feedback structure is shown in Fig.5. The equations related to the implementation of the feedback structure are similar to the feedforward structure. The basic difference between these two structures is that in the feedback structure, the estimated state is used to correct the SINS output at the time steps which the auxiliary measurements are available. Therefore, the
growth of the position error is bounded.

4. Results

Due to lack of DVL and depthmeter, underwater testing was not possible. For this reason, field experiments were carried out by a car on the ground. In this experiment, the sensor Xsens-Mti [10] and a GPS receiver from NovAtel [11] were used. The sensor Xsens-Mti and GPS antenna were mounted on the car and the data were received by a laptop as shown in Fig 6.

The MTi measures the 3D acceleration and angular rate signals and also the orientation of the vehicle, i.e. roll, pitch, and heading angles. Instead of DVL measurements, the velocity in three direction was calculated using the GPS output. The altitude provided by the GPS was used rather than depthmeter measurement.

Fig. 7 shows the car trajectory during the test. It travelled 8748 meters in approximately 51 minutes.

Figure 8 shows the position estimated in the directions of the east and north by the feedforward and feedback structures. In Fig. 9, absolute error curves are showed. Absolute error is defined as the magnitude of the difference between the reference trajectory provided by the GPS and the position estimated by the methods. According to these figures, since in the feedforward structure, the inertial system’s errors increase with time and thus the best estimation accuracy of the Kalman filter is not obtained, the estimated position deviates from the true trajectory.

In the feedback structure, due to the direct correction of the SINS output by the estimated state, the inertial system’s errors are kept small. Therefore, the position estimated by the feedback structure follows the reference trajectory with a small error.

On the other hand, it can be seen from Fig. 11 that the altitude corrected by feedback Kalman filter follows reference altitude, but the altitude corrected by feedforward Kalman diverge from the reference altitude with time.

Fig. 5: Block diagram of the integrated navigation system implemented in feedback structure

Fig. 6: Instruments mounted on the car (right) and GPS receiver and a laptop used for logging data (left)

Fig. 7: Trajectory travelled by the car (Google map).
In this test where the car has travelled approximately 8748 meters in 51 minutes, the root mean square error of the position estimated by the feedback and feedforward structures are 17.4 and 282.1 meters and the relative errors are 0.2 and 3.2 percent of the travelled distance, respectively.

In order to reduce the drift in the SINS, data fusion methods based on indirect Kalman filtering for incorporating the inertial sensors and the auxiliary sensors are utilized. In this paper, the performance of the feedforward and feedback indirect Kalman filters was investigated using real measurements. According to the result obtained from the test, the RMS error of the position estimated by the feedback and feedforward structures were 0.2 and 3.2 percent of the travelled distance, respectively. Therefore, the performance of the feedback structure is better than that of the feedforward structure. The feedforward structure operates well if the estimated inertial system’s errors are kept small. But, due to using the low cost inertial sensors, the estimated errors are increased unboundly. In the feedback structure, the inertial system’s errors are kept small. For these reason, this structure is the more robust than the feedforward structure and is essential when operating with low cost sensors.

5 Conclusion

In order to reduce the drift in the SINS, data fusion methods based on indirect Kalman filtering for incorporating the inertial sensors and the auxiliary sensors are utilized. In this paper, the performance of the feedforward and feedback indirect Kalman filters was investigated using real measurements. According to the result obtained from the test, the RMS error of the position estimated by the feedback and feedforward structures were 0.2 and 3.2 percent of the travelled distance, respectively. Therefore, the performance of the feedback structure is better than that of the feedforward structure. The feedforward structure operates well if the estimated inertial system’s errors are kept small. But, due to using the low cost inertial sensors, the estimated errors are increased unboundly. In the feedback structure, the inertial system’s errors are kept small. For these reason, this structure is the more robust than the feedforward structure and is essential when operating with low cost sensors.

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