Abstract—The dynamic response of a vehicle can be improved considerably by an active suspension, so it enjoys a growing popularity within urban cars. The vehicle vibrational models typically include many degrees of freedom, therefore the implementation of model-based feedback (feeding of state variables) control strategies in active suspension involves abundant measurements which lead to a big challenge. The observation technique is a conventional method employed to estimate the state variables and to reduce the number of output variables. As the number of measurements and corresponding sensors decrease, cost imposed to the system will decrease and the complexity of hardware implementation is also reduced. The purpose of this research is to investigate different output configurations and to determine the optimal sensory layout to achieve an ideal observer. Finally, a case study consisting of a four state quarter car model and two types of traditional sensors (acceleration and relative displacement) has been conducted. The presented algorithm takes a set of pre-defined sensors as input and analyses its various combinations. For each combination which forms a specific arrangement of outputs, the design metric is calculated through simulating assembled model of vehicle-controller-observer. This metric is a weighted sum of measure variables norm like passenger acceleration and estimation error.

Keywords—Suspension System, State Observer, Active Vibration Control, Vehicle Dynamics.

I. INTRODUCTION

The vehicle suspension system is the most important factor in shaping the dynamics of the vehicle and providing a good balance between ride and handling characteristics. Nowadays, the development of control strategies for active suspension systems is one of the major challenges in the automotive industry.

The first element of each active vibration control system is measurements and sensory units and having the state variables of system is essential for each feedback control strategies. Since the vibrational models of vehicle normally include high DOF, measuring all states requires high amount of sensors that cover a wide range of different translational and rotational measurements including quantities such as position, velocity, and acceleration. Furthermore, sophisticated data acquisition systems for processing and conditioning of measurement signals are required that increase overall costs and complexity. In mechanical systems, the measurement of acceleration is a simple task, while the state variables are normally absolute value of position and velocity which need complicated and expensive sensors. Also the measurement noise is an inevitable aspect due to wide frequency range to be measured in vehicle vibration control. Using estimation algorithms is a classical approach to overcome mentioned problems and thereby reduces the complexity and cost of the system. The purpose of this paper is to evaluate different measurements layout and to study their effects on active control of vehicle vibration, which enables the designer to select the minimum number of sensors and the most effective sensory layout. In this paper, a generic tool has been developed based on MATLAB™ graphical user interface (GUI) which contains modelling, solution and post processing of result. Employing abstract model and modular structure, gives ability to simply replace subsystems and adapt to variety of problems. This tool can be used to select optimal arrangement of measurement for a specified model and to study the estimator effects on vibrational response of vehicle. Accordingly, a model consists of vehicle dynamics, estimator structure and control has been developed. The observer model is based on the full-order Luenberger structure. Since this paper is concentrated on benchmarking between different sensory configurations, a static state feedback controller is used for suspension control. To calculate the gains of controller and Estimator, LQR and pole placement are used respectively. To demonstrate the performance and accuracy of the proposed approach, several simulations have been performed. This paper addresses the theoretical aspects of problem, so the essential data needed for a case study are collected from certified references. The case study includes a 2-DOF quarter-car model and two types of sensor including acceleration and relative displacement. The performance index consists of vertical acceleration representing ride quality and wheel normal force representing road holding. Then appropriate arrangement of sensors will be evaluated by using the provided tools.

The structure of this paper is organized as follows. The research consists of two main steps, “modeling and algorithm development” and “software implementation”. At first, a 2-DOF model is considered to describe vibrational behaviour of vehicle. Then the high-level control strategy for active vehicle suspension is designed using LQR framework. The resultant state feedback requires complete knowledge of state variables, in other words, all of them should be measured. Using an estimation template, an observer can be designed in
order to calculate state variables. The precision of observer in estimation of state variables which strongly affect suspension performance is dependent upon content and richness of provided information. This paper will provide a tool for assessment of different arrangements of sensors in order to determine proper output arrangement for an ideal observer. MATLAB GUI and Simulink modelling environment are used for providing this software tool. This tool has been developed in generic manner and let the user simply replace vehicle model and change structure of the observer. According to the research perspective which was drawn above, the theory principles related to “vehicle dynamics”, “linear observer” and “optimal control” is necessary. Since the conventional models of vehicle vibration are commonly based upon linear time-invariant (LTI) models, the corresponding theory of linear system is used.

### II. LITERATURE SURVEY

There are plenty of studies in literature related to design of observer for suspension control applications, which some of the significant ones are addressed here.

Reference [1] is one of the main works in the field of observer study in active suspension control systems. In this reference, a disturbance decoupled observer is designed that has acceptable performance when the sensor noise is small compared to the external perturbation. This paper shows that the speed of sprung and unsprung masses can’t be estimated by only measuring the relative displacement between the masses or the acceleration of them. In [2] the suspension system is considered as a disturbance affected bi-linear dynamic system. This work estimated all of the state variables only by measuring acceleration and further the observer estimation error was independent from the external disturbance.

In [3] an unknown input observer based on a quarter car model has been presented. This observer was designed in $H_\infty$ framework and it aimed to reduce the impact of uncertain road disturbance on estimated states. This observer used an accelerometer sensor on the unsprung mass in order to estimate the position and vertical velocity of sprung and unsprung masses. In [4] & [5] the $H_2$ norm of transfer function between estimation error and road input was minimized to determine the observer gain matrix, and it led to minimum effect of disturbance on observer error. In [4] the motion of vehicle while crossing the bump obstacle with trapezoidal profiles was simulated.

In [6] the Utkin reduced order observer has been used for fault diagnosis in bus suspension. Also in [7] & [8] an integrated study on the preview and estimation of state variables using reduced order observer has been conducted. This study focused on the rear suspension of a Tractor. In [9] by referring to the high cost and difficulties of force sensor installation, an observer has been utilized for estimation of the actuator force. This observer was constructed based on the measurements of body acceleration, wheel acceleration and suspension displacement.

In most of references in field of observer, the optimal state feedback (LQR method) has been used to control the system. However, some references employed other approaches such as fuzzy and sliding mode [5], back stepping [10], etc. Since disturbance is an inherent input of vehicle, the Kalman filter has been widely used to estimate state variables [11].

In [12] different configurations of measurement variables were defined, and the effects of each layout on output feedback gain were studied and it was compared with full state feedback. In this reference, a table of different output configurations was created and the corresponding gain of full state feedback and output feedback was calculated. It was shown that the lower gain corresponds to better configuration. Of course, the fully optimal control will be attained, provided that the sufficient number of feedback quantities is measured and in this situation there is no need for observer [13].

In most of the works mentioned above and other major studies in field of observer usage in suspension system, the quarter-car model has been used. Unfortunately there is not any integrated study in the context of different sensory configurations, and its impact on system performance and selection of an optimal layout. The work of [14] is the only case that considered the issue of selecting an appropriate layout of the required sensors.

On the basis of previously developed code [15] which gives ability to design study among a set of specified sensors, in this research the optimum synthesis of output variables is explored. The mentioned code has been generalized and a direct-search optimization routine is added. This capability enables designer to loop over sensors combination to obtain optimum sensory layout.

### III. MODELLING

To study the vibration behaviour of a vehicle, the model consists of rigid bodies connected by static elements of spring and dashpot is used. In addition to these elements, the force actuator is also used in active suspension system. Various models such as the quarter car model, bounce-pitch half vehicle model, and 3-DOF bounce-roll-pitch model are used in the literature for vehicle dynamics and control applications that all are based on the DOF associated with the vibration along vertical axis [16]. The goal of this research is development of a generic tool, so a general modelling template which can gather different car models under a common umbrella should be used. By adopting from approach presented in [17] and based on a force balance in the vertical direction of the vehicle, the general form of dynamic equations of the system are as follows:

$$
M\ddot{q} + C\dot{q} + Kq = B_1\ddot{x} + B_2\dot{x} + B_3f
$$

(1)

<table>
<thead>
<tr>
<th>System Dynamics</th>
<th>Measured Output</th>
<th>Cost Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{x} = Ax + Bu + B_w$</td>
<td>$y = Cx + Du + Dw$</td>
<td>$z = Gx + Hu + H_w$</td>
</tr>
</tbody>
</table>
IV. OPTIMAL LQR CONTROL

As an initial estimate to vehicle model, the LTI models are widely used which reflects vibration behaviour with a good approximation. Hence, the linear regulators broadly are used to control the suspension system [18], [19], [20], [21]. In general, the feedback control is established based on the system state variables. Optimal linear regulator problem in infinite horizon leads to Static Full State Feedback [22]. Of course it is possible to determine the state feedback in other ways such as Pole Placement. When the state variables are unavailable, observer can be used to estimate them. The output feedback can also be used in certain circumstances.

Now the formulation of optimal control for linear regulator in infinite horizon is provided. In this way, the control input is the solution of a variational problem which minimizes an objective function. Optimal regulator intends to rapidly push system variables to their set-point. The solution of this problem shapes a constant gain matrix state feedback, which preserves order of closed loop system.

\[
\begin{align*}
\text{System Dynamics} & : \quad \dot{x} = Ax + Bu \\
\text{Cost Output} & : \quad z = Gx + Hu \\
\text{Cost Functional} & : \quad J(u) = \int_0^{\infty} z'Wzdt \\
\text{Optimal Control} & : \quad u = \arg\min_x (J) = -Kx 
\end{align*}
\]

The choice of weight matrices has a drastic influence on the solution and the Bryson law [23] is used as an initial estimate. Based on this law the matrices are diagonal and its elements are selected such that the desired signals become non-dimensionalized (normally mapped to the [0, 1] interval). The mentioned formulation should be translated into MATLAB convention as follows:

\[
\begin{align*}
Q & = G'WG \\
N & = G'WH + G'W'H \\
R & = H'WH \\
\end{align*}
\]

V. OBSERVER FUNDAMENTAL

In general, an observer combines measured data (system output and those measurable inputs such as control input) with system model to estimate state variables or other signals, such as disturbance.

In this research, the structure of Luenberger observer is used in which the estimated variables are outputs of a linear dynamical system [24]. To determine the formulation of the observer, the general format of full-order observer with respect to plant dynamic (3), is considered as follows:

\[
\dot{x} = \hat{A}x + \hat{B}u + \hat{C}y
\]

The coefficients of observer are derived considering the facts that the error dynamic is independent from initial condition (asymptotical stability of internal dynamic) and from input (the measurement signals). Practically, observer poles should be several times faster than the original system (It is also mentioned in some references to 10 times, but faster rate leads to increase in system noise). By applying above conditions, observer dynamic can be written as follows:

\[
\begin{align*}
\dot{x} & = \hat{A}x + \hat{B}u + \hat{C}(y - \hat{y}) \\
\dot{\hat{y}} & = \hat{C}x + \hat{D}u 
\end{align*}
\]

The structure of Luenberger observer composed of the original system associated with a feedback from output error. This structure can be easily implemented in MATLAB and the flow graph is shown in Figure 1.

VI. IMPLEMENTATION

The final structure of the system consisting of the vehicle model, observer structure, and optimal state feedback controller is shown in Figure 1. This system forms a complex feedback structure and fortunately based on “separation principle”, it is possible to calculate either state feedback gain in absence of observer or observer gain in absence of state feedback.

The authors have been previously developed a generic tool based on MATLAB GUI which contains pre-processing (declaration of model corresponds to vehicle, road, observer, controller, etc., assignment of parameters), processing (preparing and feeding data to Simulink model, run simulation) and post processing (present and export output chart) [15]. The object-oriented approach and modular structure provide possibility to substitute alternative models and capability of adapting to variety of problems. As mentioned in previous work, it can easily be used for design study, sensitivity analysis, and optimization.

In this research, the mentioned code and a direct-search optimization routine are assembled together, which enables designer to loop over sensors combination to obtain optimum sensory layout. In order to demonstrate the performance and accuracy of the approach presented, a similar case study based on quarter-car model is conducted. Since the paper focuses on the theoretical aspect of problem, the necessary data for simulation are collected from credible references.
VII. CASE STUDY

In this section, a case study is performed on a quarter-car model to evaluate the provided tool. Despite the simplicity of the model, it reflects vibrational behaviour of body (bounce) and wheels (wheel hop) accurately and can be used as a theoretical basis to evaluate performance of different suspension systems. In the literature this model is widely used for control applications [25].

System Variables: State variables include absolute value of displacement and velocity of sprung and unsprung masses with respect to static equilibrium position. Also, the height of road profile is considered as disturbance input and actuator force as control input.

\[
x = \{x_s, \dot{x}_s, x_u, \dot{x}_u\}^T
\]
\[
u = \{f_s\}
\]
\[
w = \{x_s\}
\]

Measured Output: Measuring the absolute velocity and displacement is not a simple task and needs expensive sensors, while measuring the relative quantities and the absolute values of acceleration are simple. So the measured variables include acceleration of sprung and unsprung masses and relative displacement between these two masses. In this case study, three sensors and consequently seven outputs arrangement are available.

\[
\text{Sensors} = \{\ddot{x}_s, \ddot{x}_u, x_s - x_u\}
\]

Cost Output: In order to precisely define a metric for evaluating the performance of the suspension system, its tasks must be properly understood [16], [25]. As a result, the components of objective function can be summarized as “Ride Comfort”, “Suspension Travel”, “Road Holding” and “Active Control Energy”.

Cost output quantities are used in calculation of the cost function corresponding to a control strategy and they should be minimized. Based on mentioned indicators, these quantities are as follows:

\[
\begin{align*}
\text{Ride Comfort} & : \ddot{x}_s \\
\text{Rattle Space} & : x_s - x_u \\
\text{Actuator Energy} & : f_s \\
\text{Tire Deflection} & : x_s - x_u
\end{align*}
\]

\[
J = \int \left[ w_1 \ddot{x}_s^2 + w_2 (x_s - x_u)^2 + w_3 (x_u - x_s)^2 + w_4 f_s^2 \right] dt
\]

(8)

The above equation is a general form of the cost function which has been used in many references [9]. Additional quantities can be added to the objective function based on suspension applications and design considerations. As an example, in [26] & [27] body speed and suspension stroke have been used to reduce unwanted resonance of sprung and unsprung masses.

A. System Equation

Dynamical equations of the model and its state space form according to description of “modeling” section and presented definitions are given as follows [28]:

\[
\begin{align*}
\dot{m}_s \ddot{x}_s &= -c_s (\ddot{x}_s - \ddot{x}_u) - k_s (x_s - x_u) + f_s \\
\dot{m}_u \ddot{x}_u &= -c_u (\ddot{x}_u - \ddot{x}_s) - k_u (x_u - x_s) - k_s (x_s - x_u) - f_s
\end{align*}
\]

(9)

B. Controller Design

Based on description in “LQR” section, the feedback gain is as follows:

\[
K_p = \begin{bmatrix} -0.0273, 0.0027, 0.0273, 0.0024 \end{bmatrix}^T
\]

(10)

C. Simulation

In order to investigate the observer performance and
choose a suitable arrangement of outputs, the parameters of a sample vehicle is used as shown in Table 2.

In this case study, the transient response of active suspension system is simulated while the car is crossing over a bump. Hence, a harmonic profile similar to usual obstacles in urban streets is used [29].

\[ x_1 = h \sin \left( \frac{\pi x_2}{T} \right), \quad x_2 = \psi(x) \Rightarrow x_2(t) = \psi(Vt) \] (11)

Table 1. comparison between different sensory combination

<table>
<thead>
<tr>
<th>Layout</th>
<th>( \tilde{x}_1 )</th>
<th>( \tilde{x}_2 )</th>
<th>( \epsilon(\text{Estimation}) )</th>
<th>( J(\text{LQR}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \checkmark )</td>
<td></td>
<td>5.59</td>
<td>22.4</td>
</tr>
<tr>
<td>2</td>
<td>( \checkmark )</td>
<td></td>
<td>21.8</td>
<td>23.3</td>
</tr>
<tr>
<td>3</td>
<td>( \checkmark )</td>
<td></td>
<td>10.1</td>
<td>21.3</td>
</tr>
<tr>
<td>4</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>4.18</td>
<td>22.3</td>
</tr>
<tr>
<td>5</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>4.78</td>
<td>21.5</td>
</tr>
<tr>
<td>6</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>3.24</td>
<td>21.2</td>
</tr>
<tr>
<td>7</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>3.18</td>
<td>22.3</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td>1.44</td>
<td>11.7</td>
</tr>
</tbody>
</table>

For different combination of specified sensors, the corresponding comparison criteria presented in Table 1. The last two columns stand for integral of estimation error and cost of LQR controller.

The simulation results for full measurement and the optimum 2-sensor layout are presented in Figure 4. In Table 1, the last row corresponds to full state measurement, and as it shows the direct measuring has more suitable response than other observed cases (the error corresponds to different initial condition). Meanwhile the 2-sensor layout case may tend to good estimation, as shown. The layout 6 has acceptable estimation error which is near to 3-sensor case.

![Figure 4](image)

**Figure 4. demonstration chart for comparing plant & observer state based on layout 6 & 7**

### VIII. Evaluation & Summary

This study looks for the optimum sensory layout among a set of pre-defined sensors. To tackle with the complex nature of this problem, a tool is provided for studying the observer structure and evaluating various sensory configurations and to determine the optimal sensory layout to achieve an ideal observer.

Based on the results, it is clear that the output layout strongly affects the optimal response of system.

The introduced tool will be soon available at [30] for academic purposes. Future works include the attempt of a possible extension of proposed tool to more general strategies like adaptive and nonlinear techniques and the support of more DOF vehicle models and random-road profile. In this paper, the time response is used as the evaluation criteria, but to assess performance and robustness of closed loop, it is necessary to add frequency response measures. Also due to strong effects of disturbance on estimation error, it should be included in design of observer.
REFERENCES


[16] Theory Of Ground Vehicles (J. Y. Wong)


