Synchronized Coherent Swarming of a Collaborative Multi Agent System in Presence of Disturbance

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Abstract: In this paper, swarming of a set of agents consists of some unmanned aerial vehicles (UAVs) and some unmanned ground vehicles (UGVs) with coherent formation is studied. A decentralized robust controller is presented in order to reject the environmental disturbances and prone a rigid formation. Since the disturbance effecting UAVs is stronger than the one for UGVs, to tolerate these differences, the error of the neighbours are used by each agent as one of the entered signal of its controller to make the formation synchronized. The decentralized method improves the reliability of the performance. This algorithm is applied in simulation and the results approve the accepted performance of the proposed approach.

Keywords: Formation Control, UAV, UGV, Synchronization, Motion, Robust Control, $H_{\infty}$ Controller, Graph Theory.

1. Introduction

Advances in science make it possible to accomplish difficult task with simple individual parts. Multi agents’ theories help to make this dream become true. Nowadays Multi agent systems are increasingly used in many aspects of civilian domains, from rescue, surveillance to discovery. One of the important ability of these systems is enhancing the visibility zone by connecting the UAVs with the UGVs. However, collaboration among multi agents with different dynamics has challenges in communication and behaviour control, due to different dynamics and also different work space.

In the literature, three approaches are introduced for formation control of multi-agent system. The most common form is leader-follower in which one agent is chosen as the leader that tracks the trajectory and the other agents should keep their distances from the leader and make the predefined formation. This method is easy to understand and study. However, its main disadvantage is that the leader has no feedback from the followers and it may lead to instability when a fault occurs. Moreover, when the leader fails, the whole system will collapse [1]. The other structure is virtual. It is similar to the leader-follower structure; however the leader is virtual. Therefore, the leader never fails and the stability of the whole system is not depending on the leader [2]. The third structure is behavioural based method. It is based on the different behaviours that a single agent should perform in different situation. However, the computations of these methods are complicated [3].

The control strategy of a multi-agent system is developed based on centralized or decentralized algorithms. In central approaches, a central controller receives all required information and provides proper control signal for each agent. The mentioned algorithms rely on perfect communication and prone to fail due to connection failure or occurring fault in controller. In decentralized approaches, however, a local controller is designated for each agent and the central signals provided by using local information of agents and their neighbours [4].

In this paper, rigid formation of a team of UAVs and UGVs in presence of disturbances is studied. In [5], the problem of time varying coherent swarming is considered and is solved with the approach of differential game theorem. An inverse dynamic approach was proposed in [6] for formation of multi-agent system through a path. In [7], targeting a UGV by UAVs is considered and a Lyapunov based approach is proposed. A problem of heterogeneous agents is considered in [8]. Stability of this scenario is studied in vicinity of central mass of UGVs. [9] considered a noisy environment and designed a robust pole-placement controller in order to reject noise and also unmodeled dynamics uncertainty. In [10] a Lyapunov based controller is designed to guaranty the stable performance. In [11], the author tries to perform stable swarming in presence of disturbances by applying sliding mode control.

In [12], a synchronization method is performed to adapt the flying wings and also the [13] is improving [12] in order to make a formation under the virtual structure.

The novelty of this paper is synchronizing multi agent systems with different dynamics in presence of disturbances. In this paper a controller is proposed based on virtual leader structure to provide a rigid formation. Following the fact that UAVs usually experiment more disturbances than UGVs, a decentralized controller based on a synchronization signal is designed to achieve a predefined formation. To accomplish this goal, $H_{\infty}$ approach is employed to minimize the exogenous signals i.e. the tracking error and control signal.

The rest of this paper is organized as follows: In Section II, the dynamical model of the UGV and the UAV is studied. Then, in Section III, the problem is stated. In Section IV, a brief introduction on robust approach is illustrated. The main contribution of this paper which is design a decentralized and synchronized controller, is provided in Section V. Finally, the simulation results are presented in Section VI.
2. Dynamical Model

The Basic Problem for solving the problem of heterogeneous multi agent systems is the difference between the dynamical models of the UGVs and the UAVs. This difference makes it impossible to augment the whole system. To solve such a problem, feedback linearization method is applied to make the models similar.

In this section, the dynamics of the mobile robot and the quadrotor are given as UGV and UAV of multi-agent system and feedback linearization method is applied.

2.1 Mobile Robot

A continues time model for a mobile robot with two wheels can be defined as below [14]:

\[
\begin{align*}
\dot{x} &= \cos \theta \\
\dot{y} &= \sin \theta \\
\dot{\theta} &= \omega \\
\dot{v} &= \frac{P}{M_r} \\
\dot{\omega} &= \frac{r}{J_r}
\end{align*}
\]  

(1)

where \(x, y\) are the position of the robot, \(v\) is the linear velocity, \(\omega\) is the angular velocity, and \(\theta\) is the orientation of the robot. \(M_r\) is the mass of mobile robot, \(J_r\) is the moment of inertia, and \(P\) is the input torque of the mobile robot.

The front point of the robot can be defined as “hand point” and it can be formulated as below:

\[
\begin{align*}
x_h &= x + L_r \cos \theta \\
y_h &= x + L_r \sin \theta
\end{align*}
\]  

(2)

where \(L_r\) is the distance between the hand point and the point \(c\), which is the middle of the two tires. \(x_h\) and \(y_h\) are the coordination of the hand point. This is shown in Fig. 1.

As it has mentioned in [14], by employing inverse dynamic technique (1) and (2) can be redefined as double integrator dynamics as follows:

\[
\begin{align*}
x_h' &= u_x \\
y_h' &= u_y
\end{align*}
\]  

(3)

where the \(u_x\) and \(u_y\) are the control signals of the robot.

2.2 Quadrotor

In [15], a quadrotor is modeled by considering rigidness, and the symmetric shape of the whole body. The equation can be stated as follows:

\[
\begin{align*}
\dot{x} &= (\cos \phi \sin \theta \psi - \sin \phi \cos \theta \psi) \frac{1}{M_g} U_g \\
\dot{y} &= -\phi + (\cos \phi \cos \theta) \frac{1}{M_g} U_g \\
\dot{\phi} &= \phi \phi \left( \frac{L_r - I_z}{I_x} \right) - \frac{J_z}{I_x} \theta + \frac{1}{I_y} U_g \\
\dot{\theta} &= \phi \phi \left( \frac{L_r - I_z}{I_y} \right) - \frac{J_z}{I_y} \theta + \frac{1}{I_x} U_g \\
\dot{\psi} &= \phi \phi \left( \frac{L_r - I_z}{I_z} \right) + \frac{1}{I_y} U_g
\end{align*}
\]  

(4)

where \(x, y\), and \(z\) are the position of the quadrotor, \(\phi, \theta, \) and \(\psi\) are the orientation of it. \(I_x, I_y, \) and \(I_z\) are the body inertia, \(M_g, J_z\) and \(L_r\) are the mass, the inertia, and the length of the quadrotor, respectively. \(\Omega\) and \(U_g\) are defined as below:

\[
\begin{align*}
\{U_1\} &= b(\Omega) + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} \\
\{U_2\} &= b(\Omega) - \frac{1}{\Omega^2} + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} \\
\{U_3\} &= b(\Omega) - \frac{1}{\Omega^2} + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} \\
\{U_4\} &= b(\Omega) - \frac{1}{\Omega^2} + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} \\
\{U_5\} &= b(\Omega) - \frac{1}{\Omega^2} + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} \\
\{U_6\} &= b(\Omega) - \frac{1}{\Omega^2} + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} + \frac{1}{\Omega^2} \\
\{\Omega\} &= \Omega + \Omega - \Omega - \Omega - \Omega
\end{align*}
\]

where \(\Omega\) is the inputs of the motors, \(b\) is the trust factor, and \(d\) is the drag factor.

The above system can be modeled as a double integrator with a Lyapunov function to stabilize the orientation loop, and an inverse kinematic. Therefore, the quadrotor can be reformulated as follows:

\[
\begin{align*}
x_h' &= u_x \\
y_h' &= u_y \\
z_h' &= u_z
\end{align*}
\]  

(5)

where the \(u_x, u_y,\) and \(u_z\) are the control signals of the quadrotor.

3. Problem Statement

In this section, the main problem and required definitions are given.

Consider \(m\) agents with double integrator model as given in (3),(5). The state equation of each agent can be formulated as follows:

\[
X = AX + BU
\]  

(6)
where \( \mathbf{x} = [x_1, y_1, z_1, \dot{x}_1, \dot{y}_1, \dot{z}_1]^T \) if the agent is mobile robot and \( \mathbf{x} = [\dot{x}, \dot{y}, \dot{z}, \dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z]^T \) if the agent is quadrotor, and \( \mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \), and \( \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \). In this paper, the goal for each agent, is keeping a predefined distance to its neighbors and also the virtual leader in order to make a rigid formation. To achieve this goal, it is important to investigate the connection topology among the agents. A usual definition of the connection structure is using graph theory. If each agent is considered as a node, then the connection topology among the agents can be described by a simple graph [17]. A weighted digraph (directional graph) \( \mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathbf{D}) \) is considered with \( n \) agents and the set of nodes \( \mathcal{V} = \{1, 2, ..., n\} \), the set of connection links \( \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \), and a weighted adjacency matrix \( \mathbf{A} = [a_{ij}] \in \mathbb{R}^{n \times n} \) with non-negative elements. The set of neighbors of node \( i \) is shown by \( \mathcal{N}_i = \{ j \in \mathcal{V} | (i, j) \in \mathcal{E} \}. \) If \( \sum_{j \in \mathcal{N}_i} a_{ij} = \sum_{j \in \mathcal{N}_j} a_{ji} \) for all \( i = 1, 2, ..., n \), the digraph \( \mathcal{G} \) is called balanced [16].

\( D \) is a an incident matrix whose elements are 1, -1, or 0. Its elements is the connection between nodes of \( \mathcal{V} \) and edges of \( \mathcal{E} \). The \( ij \)th element of \( D \) is equal to 1 if the node \( i \) is the head of the edge and \( j \) belongs to the \( \mathcal{N}_i \), -1 if the node \( i \) is the tail of the edge and \( j \) belongs to the \( \mathcal{N}_i \), and 0 otherwise [5].

The main contribution of this paper is to make a synchronized rigid formation under different disturbances, which is imposed to the system due to presence of different type of vehicles. It is clear when there are UAVs and UGVs in a multi agent collaboration, the environmental disturbance such as winds, has affect on UAVs more seriously than the UGVs. Therefore, it is important to consider a strategy to synchronize their movements.

The main objective is to minimize the tracking error between each agent and the virtual leader:

\[
\text{Minimize } \mathbf{e}_f = \mathbf{x}_f - \mathbf{x}_L \tag{7}
\]

where \( \mathbf{e}^v_f = \mathbf{x}^v - \mathbf{x}_L \), in which \( \mathbf{x}_L \) is the state vector of the virtual leader.

4. Robust Controller

One of the main difficulties of control of multi agents systems is having a rigid formation in presents of environmental disturbances. This disturbance can ruin the formation an also can also make the system instable. To avoid such a problem, robust control is proposed.

Robust control approach is based on minimization of a transfer function of the system in presence of uncertainties, disturbances, and/or measuring noise. The basic concept of robust controller design is based on Linear Fractional Transformation (LFT). For this purpose, the system should be expressed to a standard form that is illustrated in Fig. 2:

\[
P = \begin{bmatrix} A & B_1 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \tag{13}
\]

where \( P \) is the matrix of system, \( K \) is the controller, \( A \) is the whole possible uncertainty, \( w \) is an input vector consist of noise, disturbance, and the reference signal, \( z \) is exogenous signal consist of tracking error, \( u \) is the control signal, and \( y \) is the measured signal. \( v \) and \( \eta \) are the input and output of the uncertainty block [18].

The main objective of designing an \( H_\infty \) controller is to minimize the transfer function from \( z \) to \( w \):

\[
\| \mathbf{z} \|_{\infty} \leq \gamma \tag{6}
\]

\[
\| \mathbf{w} \|_{\infty} \leq \frac{1}{\gamma} \tag{7}
\]

in which \( \gamma \) should be found so that the closed loop system is stable in presence of the admissible \( \Delta \) [18].

5. Controller Design

In Section III, the basic concepts are illustrated. Then, by explaining \( H_\infty \) approach, it is time to define our problem in a Standard form in order to use robust \( H_\infty \) controller. Indeed, the most important step to struggle with a robust problem is to define the input and exogenous signal in a proper way.

Since each agent can be written in a double integrator form, then by considering the disturbance effect on the input signal, the dynamics of each agent can be stated as follows:

\[
\dot{x}_f = u_f + \omega_{nf} \tag{10}
\]

where \( \mathbf{x}_f \) is the position of the agent \( f \), \( u_f \) is the control signal of the agent \( f \), and \( \omega_{nf} \) is the disturbance that effects the \( f \)th agent. Then the error vector can be defined as:

\[
\mathbf{e}_f = \mathbf{x}_f - \mathbf{x}_L \tag{11}
\]

By taking time derivative of (11) two times, one can get:

\[
\dot{\mathbf{e}}_f = \dot{\mathbf{x}}_f - \dot{\mathbf{x}}_L \tag{12}
\]

Then substituting (10) from (12) yields:

\[
\dot{\mathbf{e}}_f = u_f + \omega_{nf} - \mathbf{x}_L \tag{12}
\]

Now by defining \( \omega_{nf} = u_f - e_f \), one can get:

\[
\dot{\mathbf{e}}_f = \omega_{nf} \tag{12}
\]

Now, the system can be written in the standard form of (13). (14), (15), and (16) illustrate the whole system interaction [18]:

\[
P = \begin{bmatrix} A & B_1 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \tag{13}
\]
where $u$ is the input signal of the plant, $w$ is the input signal of the controller, and $z$, $x$, and $w$ are defined as below:

\[
\begin{align*}
  z &= \left[ z^1 \ z^2 \right] \\
  x &= \left[ x^1 \ x^2 \right] \\
  w &= \left[ w^1 \ w^2 \right]
\end{align*}
\]

where $e_{\text{ref}}$ is the error vector of the agents in neighborhood which is defined as (17):

\[
\begin{align*}
  e^u &= \sum_{i \in \mathcal{N}} a_{ij} e^u_j \\
  e^\phi &= \sum_{i \in \mathcal{N}} a_{ij} e^\phi_j \\
  e_{\text{ref}} &= \begin{bmatrix} e^u \ e^\phi \end{bmatrix}
\end{align*}
\]

where $e_{\text{ref}}$ is the synchronization signal which guarantees acceptable performance of the system. The matrix of the standard form of $H$ can be easily written as follows:

\[
\begin{align*}
  x^1 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\
  x^2 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\
  z &= \begin{bmatrix} z^1 \\ z^2 \end{bmatrix} \\
  y &= \begin{bmatrix} y^1 \\ y^2 \end{bmatrix}
\end{align*}
\]

1. $H$ is stabilizable and $C_1$ is detectable.
2. $H$ is stabilizable and $C_2$ is detectable.

These two conditions guarantee that the Hamiltonian matrixes of (18) and (19) are belonged to the domain of Riccati equation of (20) and (21):

\[
\begin{align*}
  H^u &= \begin{bmatrix} A & -B_1 B_1^* - B_2 B_2^* \\ C_1 C_1^* & -A^* \end{bmatrix} \\
  J^u &= \begin{bmatrix} A^* & -B_1 B_1^* - C_2 C_2^* \\ C_1^* C_1 & -A \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
  A^* x_{\text{ref}} + X_{\text{ref}} A + C_1^* C_1 + A^* B_1 = 0 \\
  A^* Y_{\text{ref}} + Y_{\text{ref}} A + B_1 B_1^* + A^* C_1 Y_{\text{ref}} = 0 \\
  -Y_{\text{ref}} C_2 C_2^* Y_{\text{ref}} = 0
\end{align*}
\]

where $x_{\text{ref}}$ is the answer of the Riccati equation of (20) and $y_{\text{ref}}$ is the answer of the Riccati equation of (21).

**Theorem:** There is an admissible controller such that $[y_{\text{ref}}]_{\infty} < \rho$ if the following three conditions hold [18]:

1. $H^u \in \text{dom}(Ric)$ and $x_{\text{ref}} = \text{Ric}(H^u) > 0$
2. $J^u \in \text{dom}(Ric)$ and $y_{\text{ref}} = \text{Ric}(J^u) > 0$
3. $\rho [y_{\text{ref}}]_{\infty} < \rho^2$

where $\rho(\Delta) = \max_{\Delta_{\text{ref}}} \| \Delta \|$ is the spectral radius. When these conditions hold, one such optimal controller is [18]:

\[
K_{\text{ref}}(s) = \begin{bmatrix} \frac{\Delta}{s + \lambda} & -E_{\text{ref}} L_{\text{ref}} \\ 0 & 0 \end{bmatrix}
\]

where $\Delta = A + \gamma^{-1} B_1 B_1^* X_{\text{ref}} + B_2 R_{\text{ref}} + L_{\text{ref}} C_2$, $R_{\text{ref}} = -B_2^* X_{\text{ref}}$, $L_{\text{ref}} = -Y_{\text{ref}} C_2^*$, $S_{\text{ref}} = (I - \gamma^{-1} Y_{\text{ref}} Y_{\text{ref}})^{-1}$.

**6. Simulation**

In this section, the proposed formation controller (22) is simulated for a multi agent system, consist of a virtual leader (VL), 3 mobile robot, and 3 quadrotors. The rigid formation and the relation between the agents are depicted in Fig. 3.

![Fig. 2: The desired formation](image)

Every agent receives the error of position and velocity of its neighbours. The virtual leader (VL) sends its position and velocity to all of the agents.

The parameters are chosen as shown in Table. I.

**TABLE I: The Numerical Values of the Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>2.53</td>
</tr>
<tr>
<td>$l_1$</td>
<td>0.1676</td>
</tr>
<tr>
<td>$l_2$</td>
<td>0.1686</td>
</tr>
<tr>
<td>$l_3$</td>
<td>0.29743</td>
</tr>
<tr>
<td>$g$</td>
<td>9.81</td>
</tr>
<tr>
<td>$M_e$</td>
<td>1</td>
</tr>
<tr>
<td>$l_4$</td>
<td>1</td>
</tr>
</tbody>
</table>

In order to have such formation, the incident matrix $D$ is defined as below:

\[
D = \begin{bmatrix} 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}^{T}
\]
Note that the virtual leader is considered as a node, too. The desired distance between the agents and the VL are as follows:

\[
D_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}; D_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}; D_4 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\]

\[
D_5 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}; D_6 = \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \end{bmatrix}; D_7 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\]

The simulation is shown in Fig. 4 to Fig. 9. The disturbance is a sinusoidal signal with the magnitude of 0.1:

The control signals which are produced by the controllers are shown in Fig. 10 and Fig. 11:
As it is clear in the simulation results in Fig. 4 to Fig. 9, the desired formation can be obtained through the controller designed in Section V. It is obvious that the synchronization signal can help the agents to reach the desired formation faster and more smoothly. As it is clear in the results, when there is environmental disturbance, the synchronization approach can perform better. In addition, according to the control signals, the disturbances are up to 50 percent of the entering signals, which means, with good approximation, the system is robust to disturbance.

4. Conclusion

In this paper, an $H_{\infty}$ controller is proposed in order to guide the agents with a rigid formation in a swarming path. Then by defining a synchronization signal which is applied to the neighbours of the agents, acceptable performance in presents of disturbance is guaranteed. The transient time to reach the predefined formation decreases incredibly by this approach as it is easily figured out in the simulation results.

References