

## A Trapezoidal Membership Function Block for Fuzzy Applications

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**Abstract:** A new Switched Capacitor Membership Function circuit, which is based on an offset insensitive SC amplifier and presenting trapezoidal-shape transcharacteristics is proposed. The circuit is realized in a 0.35-um standard CMOS technology and simulation by Hspice is carried out.

**Keywords:** Fuzzy rule, Membership degree, Membership Function Block, Trapezoidal MFB.

### 1 Introduction

Over recent years, due to inherent ability of fuzzy signal processing in expressing complex control laws by using simple systems of rules, application of fuzzy systems has become very popular [2-3]. Moreover, in some applications, fuzzy control has been proved to achieve better performances with respect to conventional methods.

A large number of basic building blocks [5-8] or fuzzy systems, suitable for a VLSI digital or analog implementation, are contained in literature. In particular, digital implementations have the advantage of both simple design procedure and good accuracy but they require a larger silicon area with respect to equivalent analog ones. This area occupation increases if the circuit has to communicate with the real world because A/D and D/A converters are mandatory for both inputs and outputs. On the other hand, analog implementation has better performances in terms of silicon area and speed requirements, but they need a large effort during the design.

The Switched Capacitor (SC) technique appears to constitute a good compromise between speed, accuracy, area and effort of design. In fact, although its speed limitation compared to a continuous-time implementation, it provides the benefits of both small area occupation and good accuracy. Moreover, the efforts during the circuit realization can be minimized by using macro cells of fundamental blocks (opamps, switches,

capacitive arrays, etc.), thus simplifying the layout process.

A fundamental cell in fuzzy systems is the Membership Function Block (MFB) which returns the membership degree of a variable with respect to a given fuzzy set. In this paper, modifying a triangular-shape Membership Function Block, a trapezoidal-shape Membership Function Block in SC technique is presented. The proposed MFB is realized in a 0.35um standard CMOS technology and simulation results are given.

### 2 The Transcharacteristic equation

A trapezoidal MFB, shown in fig.1 can be mathematically represented by the following equations:

$$V_{out} = \begin{cases} 0 & V_{in} < V_{t1} - \frac{V_{max}}{\alpha} \\ \alpha(V_{in} - V_{t1}) + V_{max} & V_{t1} - \frac{V_{max}}{\alpha} < V_{in} < V_{t1} \\ V_{max} & V_{t1} < V_{in} < V_{t2} \\ -\alpha(V_{in} - V_{t2}) + V_{max} & V_{t2} < V_{in} < V_{t2} + \frac{V_{max}}{\alpha} \\ 0 & V_{in} > V_{t2} + \frac{V_{max}}{\alpha} \end{cases} \quad (1)$$

Referring fig. 1 parameters  $a$  and  $V_{max}$  represent the slope of the transcharacteristic and its maximum value respectively, while  $V_{t1}$  and  $V_{t2}$  show the interval during which the trapezoid reaches its maximum value.

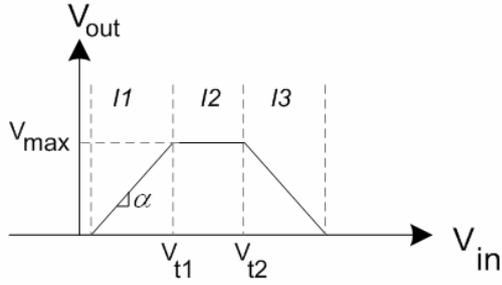


Figure 1: Transcharacteristic of a MFB

The transcharacteristic expressed by (1) can simply be implemented by performing a max function between zero and trapezoid around the vertex  $V_{in}$ . This implies that (1) can be implemented in two consecutive steps. In the first step a trapezoidal transcharacteristic is extracted (2a) and initial output of  $V_{out}^*$  is obtained, where during the second step the max operation between zero and  $V_{out}^*$  is performed.

$$V_{out}^* = \begin{cases} \alpha(V_{in} - V_{t1}) + V_{max} & V_{in} < V_{t1} \\ V_{max} & V_{t1} < V_{in} < V_{t2} \\ -\alpha(V_{in} - V_{t2}) + V_{max} & V_{in} > V_{t2} \end{cases} \quad (2a)$$

$$V_{out} = \max(V_{out}^*, 0) \quad (2b)$$

Note that the function expressed in (2b) can be utilized by the part of the circuit which calculates the activation degree of fuzzy rule. Indeed the activation degrees are generally calculated by extracting the minimum of two or more membership degrees. Assuming that there already exists a proper minimum block, conversion of this block into a maximum block is simple. Fig. 2 clarifies how this conversion can be performed. Suppose that the activation degree,  $\mu_k$ , for the generic rule,  $k$ , is the minimum of two MFB outputs,  $MFB_i$ ,  $MFB_j$  and  $V_{max}$ . This operation can be carried out as illustrated in Fig. 2a, where Fig. 2b illustrates a better and equivalent implementation.

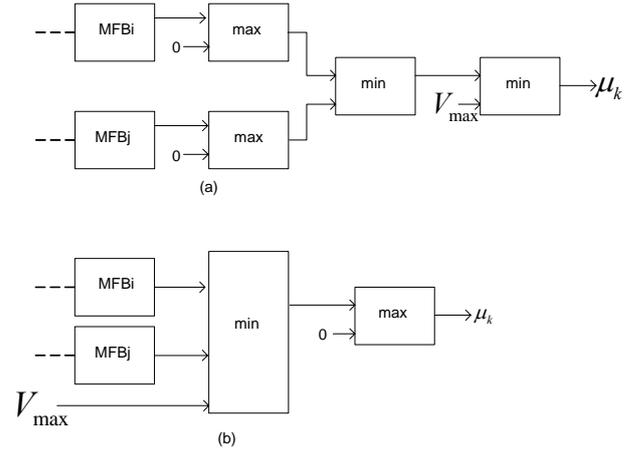


Figure 2: Fuzzy rule extraction. (a) Basic idea. (b) Practical implementation.

### 3 The SC MFB

Thanks to special merits of switched capacitor circuits, realization of the MFB is proposed by this technique.

#### 3.1 The difference SC amplifier

The MFB uses the amplifier in Fig. 3. and

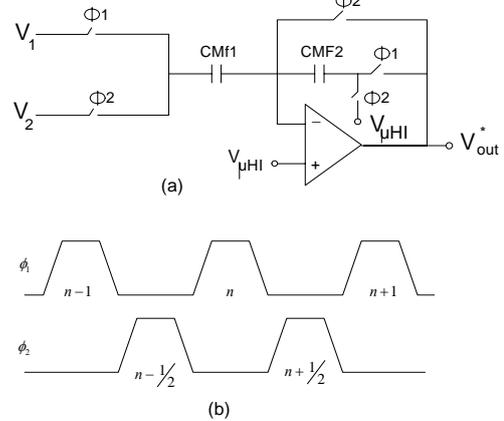


Figure 3: (a) The two input SC amplifier. (b) Clock phases

works as follows. Assuming as positive the charge in the top plates of both capacitors, during phase  $\phi_2$ , in z-domain

$$Q_{MF1} z^{-\frac{1}{2}} = C_{MF1} (V_{\mu HI} - V_2 z^{-\frac{1}{2}}) \quad (3a)$$

$$Q_{MF2} z^{-\frac{1}{2}} = 0 \quad (3b)$$

and during phase  $\phi_1$

$$Q_{MF1} = C_{MF1}(V_{\mu HI} - V_1) \quad (4a)$$

$$Q_{MF2} = C_{MF2}(V_{\mu HI} - V_{out}^*) \quad (4b)$$

Since the total charge during  $\phi_2$ , must be equal to the total charge during  $\phi_1$  we get :

$$V_{out}^* = \frac{C_{MF1}}{C_{MF2}} \left[ V_2 z^{-\frac{1}{2}} - V_1 \right] + V_{\mu HI} \quad (5)$$

As is well known, the amplifier in Fig. 3 is insensitive to the OTA offset. Indeed, by assuming  $V_1$ ,  $V_2$  and  $V_{\mu HI}$  equal to zero and the offset modelled as a signal  $V_{OS}$  applied to the non-inverting input, we get for  $V_{out}^*$  :

$$V_{out}^* \approx -\frac{1}{A} \left( \frac{C_{MF1}}{C_{MF2}} \right) v_{os} \quad (6)$$

Since a typical offset is in the order of a few millivolts, for a gain of about 60 dB, this contribution is negligible.

### 3.2 Triangular MFB

The MFB that realizes the transcharacteristic described by (7) is shown in Fig. 4.

$$V_{out}^* = \begin{cases} \alpha(V_{in} - V_c) + V_{\mu HI} & V_{in} \leq V_c \\ -\alpha(V_{in} - V_c) + V_{\mu HI} & V_{in} \geq V_c \end{cases} \quad (7)$$

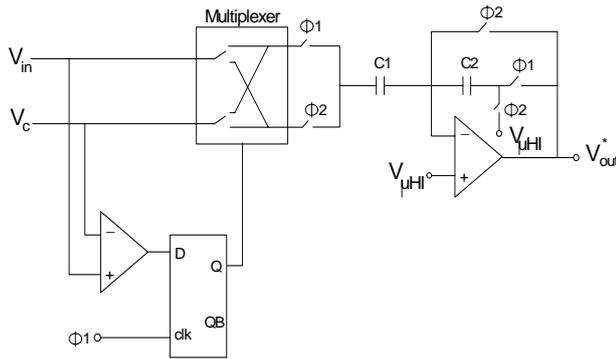


Figure 4: The MFB of Triangular Transcharacteristic

It is made up of the amplifier in Fig.3 and an analog multiplexer driven by a voltage comparator which evaluates whether the  $V_{in}$  is greater or smaller than  $V_c$ . The flip flop is incorporated to protect from glitches that arise from comparator output when  $V_{in}$  is too close to  $V_c$ . Depending on the sign of the difference between  $V_{in}$  and  $V_c$  if the

flip flop output,  $Q$ , is logic high the charge of the capacitor  $C_1$  will depend on  $V_{in}$  during phase  $\phi_1$  and on  $V_c$  during  $\phi_2$ . Respectively if  $Q$  is logic low then the  $V_{in}$  and  $V_c$  are exchanged providing a sign inversion in the charge stored by  $C_1$ . Taking into account (5) we obtain  $V_{out}^*$  as:

$$V_{out}^*(z) = \begin{cases} \frac{c_1}{c_2} (V_{in}(z^{-1/2}) - V_c) + V_{\mu HI} & V_{in} \leq V_c \\ -\frac{c_1}{c_2} (V_{in}(z) - V_c) + V_{\mu HI} & V_{in} \geq V_c \end{cases} \quad (8)$$

### 3.3 Trapezoidal MFB

Since in a trapezoidal membership function we have 3 intervals to define the trapezoid, as shown in Fig. 1, we use irregular phases in the switched capacitor circuit. During interval  $I_1$ , when  $V_{in}$  is smaller than  $V_{I1}$  the  $Q_1$  is high, Fig. 5, and the charge of the  $C_1$  will depend on the voltage of node  $in_0$  during  $\phi_1$ , that is  $V_{in}$ , and on  $V_c$  during  $\phi_2$ .

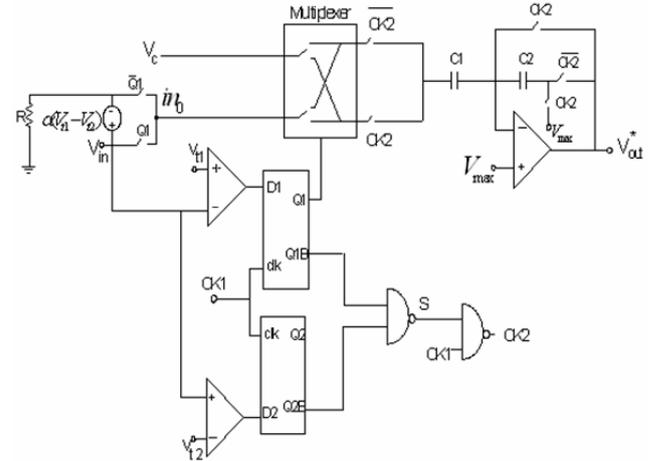


Figure 5: The MFB of Trapezoidal Transcharacteristic

Throughout the interval  $I_2$ , where  $V_{in}$  is between  $V_{I1}$  and  $V_{I2}$ ,  $Q_1$  is low the charge of the  $C_1$  depends solely on the  $V_c$ . During this interval no complementary clock phases are active anymore since the  $CK_2$  is zero during this interval, Fig. 6(c). Also note that the  $CK_1$  is the reference clock of the circuit and  $CK_2$  is obtained by the NAND of the  $CK_1$  and  $S$ .

During the third interval,  $I_3$ ,  $Q_1$  is low and the charge of the  $C_1$  will depend on  $V_c$  during phase  $\phi_1$  and  $V_{in} - \alpha(V_{I1} - V_{I2})$  during phase  $\phi_2$ .

## 4 Simulation Results

The described MFB is simulated in a  $0.35\mu\text{m}$  standard CMOS technology with  $V_{in}=0.6\text{v}$ ,  $|V_{tp}|=0.75\text{ v}$ ,  $K_n=74\ \mu\text{A}/\text{V}^2$  and  $K_p=25\ \mu\text{A}/\text{V}^2$ . The clock signal was set to 4MHz and the power supply to 3.3 v. The amplifier was implemented with a folded cascade OTA. With a 1-pF capacitive load, it has a 60 degree-phase margin, at the transition frequency of 12 MHz. A circuit simulation is reported in Fig. 6. The vertex of the membership function,  $V_c$  is 1.6V and the maximum value of the transcharacteristic,  $V_{max}$ , is 2V.  $V_{t1}$  and  $V_{t2}$  are set to 1.65V and 2V respectively. The slope of the triangular shape (that is the ratio between C1, and C2) is equal to 1. The correct value is available only during phase  $\phi_1$ , because, during  $\phi_2$ , the output goes to  $V_{max}$ . Fig. 6(a) and 6(b) show the  $V_{out}^*$  and the final output after a sample-and-hold block respectively. Fig. 6(c) shows the CK2 that is used in Fig. 5.

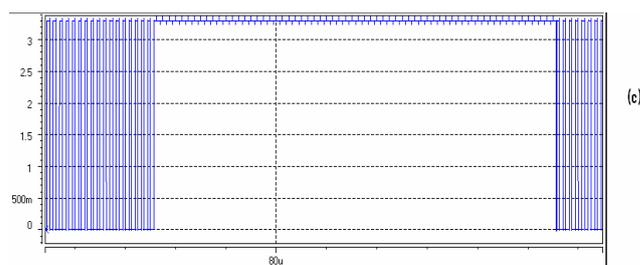
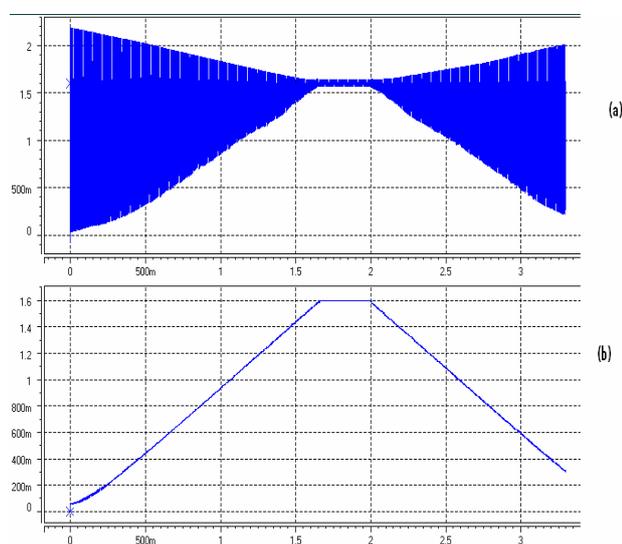


Figure 6: simulation results a) output waveform b)sampled and held output c) CK2 waveform

## 5 Conclusion

A new Switched Capacitor Membership Function circuit, which is based on an offset insensitive SC amplifier and presenting trapezoidal-shape transcharacteristics was proposed. The circuit was realized in a 0.35-um standard CMOS technology and demonstrated by Hspice simulation.

## 6 References

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