**SOLVING INVERSE HEAT CONDUCTION PROBLEM BY NELDER-MEAD SIMPLEX SEARCH METHOD**

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**ABSTRACT**

In this paper Nelder-Mead (NM) simplex search method combines with least squares method for the determination of temperature in an inverse heat conduction problem (IHP). The performance of NM is established with an examples of IHP. Numerical results are obtained by implementation NM on 2.20GHz clock speed CPU.

**Keywords:** Nelder-Mead simplex search method, The least squares method, inverse heat conduction problem.

1. **INTRODUCTION**

Most phenomena in the real world can be described through heat conduction equations which have attracted lost of attention from among scientists.

For example, consider that a room begins to warm up because the sun is shining into it. Apparently, the physical property of interest here is the temperature which is a function of the time and space. The temperature variations can be represented by a heat conduction problem (HCP) as the governing equation.

In heat conduction problems, there is a problem in the main equation and its conditions. This form of problem is called inverse problems.

On the contrary there is another category of problems wherein, in addition, there is one unknown factor in the problem equation. This type of problem is called direct problem. However, there is another type of problems wherein, in addition to the unknown main factor, there are other characteristics in the equation and its conditions. This form of problem is called inverse problems.

In this paper Nelder-Mead (NM) simplex search method combines with least squares method for the determination of temperature in an inverse heat conduction problem (IHP). The purpose of the study was to find an unknown boundary condition in IHCPS by using over-specified condition. Problem (1.1) can be solved in least-square sense and a cost function can be defined as a sum of squared differences between measured temperatures and calculated values of $U(x,t)$ by considering guesses as the estimated values of $q(t)$.

$$f(Guesses estimated values of q(t)) = \sum_{j=1}^{m} (U(a,t_j) - s_j)^2,$$

where $U(a,t_j)$, $j = 1,2,3,\ldots,m$, are calculated by solving the direct HCP. To this end, prior guess for $q(t)$ can be considered. In addition, $s_j = s(t_j)$, $j = 1,2,3,\ldots,m$, are measured temperatures at $x = a$. Equation (1.2) must be minimum so that the optimal solution of $q(t)$, can be found.

2. **NELDER-MEAD SIMPLEX SEARCH METHOD FOR SOLVING IHCP**

This simplex search method was first proposed by Spendley, Hext, and Himsworth, and was later refined by Nelder and Mead [2]. Their method is one of the most efficient pattern search methods currently available. This method is a derivative-free line search method particularly designed for traditional unconstrained minimization scenarios, such as the problems of nonlinear least squares, nonlinear simultaneous equations, and other types of function minimizations. In this method, for $N$ vertices of an initial simplex, the cost function for each vertex is first evaluated. Then, the previous vertex is replaced by a better newly reflected point, which can be approximately located in the negative gradient direction. In the minimization problem with three initial simplex vertices, the method can be mentioned as follow[3]:

- $x_h$: Vertex with highest cost function value.
- $x_c$: Vertex with the second highest cost function value.
- $x_i$: Vertex with lowest cost function value.
- $x_c$: The centroid of vertices except $x_h$.

1. **Reflection.** Reflect $x_h$ (figure 1) and find $x_0$ such that

**Figure 1:** Reflection $x_h$ toward $x_0$. 
If \( f(x_1) < f(x_3) \), replace \( x_h \) by \( x_3 \) and return to step 1.

3. Expansion. If \( f(x_0) < f(x_1) \) then expansion operation makes \( x_{00} \) (figure 2). Based on the function value, replace \( x_h \) by either \( x_0 \) or \( x_{00} \). More specifically
   (a) If \( f(x_{00}) < f(x_3) \), replace \( x_h \) by \( x_{00} \)
   (b) If \( f(x_{00}) > f(x_3) \), replace \( x_h \) by \( x_0 \)

Then, return to step 1.

4. Contraction. If \( f(x_0) > f(x_1) \), then the contraction operation of \( x_{00} \) is performed (done) by considering the following two cases:
   (a) If \( f(x_0) < f(x_h) \) (figure 3) find \( x_{00} \) such that
      \[ x_{00} = \frac{1}{2} x_0 - \frac{1}{2} x_c \]

   (b) If \( f(x_0) \geq f(x_h) \) (figure 4) find \( x_{00} \) such that
      \[ x_{00} = \frac{1}{2} x_h - \frac{1}{2} x_c \]

There are two more cases to consider in using 4.a or 4.b

c. If \( f(x_{00}) < f(x_h) \) and \( f(x_{00}) < f(x_0) \) then replace \( x_h \) by \( x_{00} \) and return to step 1.
d. If \( f(x_{00}) \geq f(x_h) \) or \( f(x_{00}) > f(x_0) \) then reduce size of simplex by halving distances from \( x_1 \) and return to step 1.

The process terminates when either the number of iterations has exceeded a preset amount, or the simplex size is smaller than a given value.

In this paper, an attempt was made to consider vectors that estimate unknown \( q(t) \) as the vertices of the simplex to be put in equation (1.2) as the cost function. The initial vertices were randomly generated. In order for the unknown \( q(t) \) to be found, the final vertex was interpolated at the end of NM simplex search method.

3. MAIN RESULTS

An example may demonstrate, numerically, some of the results for the unknown boundary condition in the reverse problem (1.1) by NM simplex search method. In this example, let us consider the following one-dimensional IHCP,

\[ U_t(x,t) + U(x,t)U_x(x,t) = U_{xx}(x,t), \quad 0 < x < 1, \quad 0 < t < t_M \]  

\[ U(x,0) = \frac{1}{2} - \frac{1}{2} \tanh\left(\frac{x}{4}\right), \quad 0 \leq x \leq 1, \]  

\[ U(0,t) = \frac{1}{2} - \frac{1}{2} \tanh\left(\frac{t}{8}\right), \quad 0 \leq t \leq t_M, \]  

\[ U(1,t) = q(t), \quad 0 \leq t \leq t_M, \]  

and the overspecified condition

\[ s(t_j) = U(0.5,t_j), \quad t_j = 0.05 \times j, \quad j = 0, 1, 2, \ldots, 20. \]  

where the unknown function is the continuous function \( q(t) \) as, \( q(t) = \left( \frac{1}{2} - \frac{1}{2} \tanh\left(\frac{t}{8}\right) \right) \). Figure 5 present exact and numeric \( q(t) \) for 100 iterations by implementing Nelder-Mead simplex search method for above example.

![Figure 5: Exact and numeric q(t) for 100 iterations by implementing Nelder-Mead simplex search method at example.](image-url)

4. REFERENCES
